

Discussion Paper

Risk-adjusted Valuation of the Real Option to Invest

December 2014

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Risk-Adjusted Valuation of the Real Option to Invest

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Abstract

This paper resolves the conceptual ambiguity of real option value and derives a model using risk-adjusted discount rates that can be applied to value the option to invest in a project. The approach adopts stochastic revenue and costs which provide a general solution with the added virtue of applicability. We found the option value arises from the difference between an individual investor and the market in financing efficiency and risk preferences. Investors' taking on idiosyncratic risks are crucial to obtaining the real option value; hedging project risks can significantly reduce the associated real option value.

Keywords: real option, decision making, investment opportunity, geometric Brownian motion, option to invest, discounted cash flow, change of measure, risk tolerance, risk aversion, idiosyncratic risk, gold mine

JEL Classification: C44, D81, G11, G30

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1 Introduction

This paper starts from the basis that all investment projects are perceived to contain certain real options. Some of these options are widely recognised and are therefore fully priced within the value ascribed to the project value. Other options are however not publicly recognised and it is in these private options that positive value can be found. This view reconciles the argument put forward by Myers (1977), Berk, Green, and Naik (1999) and others, that the market value of an asset should incorporate the value of the real option attached to the asset, with the influential work of Copeland and Antikarov (2001); Henderson (2007) – and much of the applications literature since – which argues that the real option value is additional to the market value of the underlying asset.¹

We argue that the value of a real option has two components – that perceived by the market and that perceived by the individual investor. The first component is incorporated in the equilibrium project value, in line with Myers (1977) and others, whereas the second is consistent with Copeland and Antikarov (2001) and Henderson (2007) and we refer to it as the “add-on” option value. It follows that a project value which incorporates only the first component merely provides the market return – that is, the investor would obtain a zero net present value (NPV) on the project. It is only the “add-on” value which denotes the value of the individual investor’s opportunity to obtain a profit (acquiring a positive NPV) from “buying” the project.

Hence, in our framework, this part of the option value is entirely subjective: the receipt of the add-on value can be considered as a reward to the individual investor who, in order to obtain the abnormal return, needs to execute an investment strategy that is optimised according to his *own* financing efficiency and risk preferences, instead of those of the market representative investor. Only when the individual investor has identical financing efficiency and risk preferences as the

¹The real option that is referred to here only represents the real option that has not been contracted independently with the underlying project, such as an opportunity of an investor to take over a project at its market price whenever the take-over decision is made. A contracted real option would be an official statement about the right of an investor to execute some investment strategy associated with the project at some pre-determined costs, such as a warrant for developing a piece of undeveloped land or an agreement between the buyer and the seller of a project that allows the buyer to take over the project from the seller at a stated price. The contracted real option would be very similar to a untradable or extremely illiquid financial option and it obviously is much less common than the real option not yet contracted.

market representative investor, should this “add-on” value be zero. This is indeed consistent with [Grenadier \(2002\)](#) who discusses a special scenario of this case, that is, when the market is perfect. But, by contrast with [Copeland and Antikarov \(2001\)](#) and [Henderson \(2007\)](#) who fix the option’s strike price (which, we show, implies that the add-on value does not converge to zero in a perfect market), we allow the strike price to reflect the stochastic view of the market about the project value. Furthermore, we do not make any assumption on the replicability of the real option or its underlying project and, innovatively, model the add-on value under a risk-adjusted measure, unique to each individual investor rather than to all investors in the market. Under this measure which depends on the investor’s financing efficiency and risk preferences, all future project values are presented in present value (PV) terms. Hence, our approach is consistent with the traditional valuation of the underlying project (*eg.* the discounted cash flow, DCF, framework) and does not require the assumption of a complete market for the project or that an individual investor is endowed with a utility function.

We are the first to incorporate the expected project return in the real option valuation and investigate its impact on the add-on option value. Moreover, following the utility-based models of [Hugonnier and Morellec \(2007a,b\)](#); [Henderson \(2007\)](#); [Miao and Wang \(2007\)](#), we study the impact of other parameters on the add-on option value, including the project value volatility and idiosyncratic risk effects. Importantly, we show that the add-on value rises when the project return expected by the individual investor diverges from the expected return implied by the market value of the project, indicating that the add-on value arises from the difference between the individual and market representative investors in terms of financing efficiency and risk preferences. This then underlies the non-monotonic effects of both the idiosyncratic risks and the investor’s risk aversion on the add-on option value. These effects illustrate that it is not necessarily true that less risk-averse investors receive greater profits from bearing idiosyncratic risks.

Moreover, the idiosyncratic risk effect can be negative as identified by [Hugonnier and Morellec \(2007a,b\)](#); [Henderson \(2007\)](#); [Miao and Wang \(2007\)](#). This usually happens if the investor is extremely inefficient or risk averse. More generally, the idiosyncratic risk is positive,

implying that real options are not necessarily opportunities for the investor to avoid exposure to idiosyncratic risks. The positive idiosyncratic risk effect can occur even when we set negative market premia for idiosyncratic risks involved in the project; this setting is concordant with [Ang, Hodrick, Xing, and Zhang \(2006\)](#) who find significant negative market premia for idiosyncratic risks in capital markets. It therefore is implied that while the market discourages taking on idiosyncratic risks, the investor has a significant opportunity to profit from bearing such risks.

The add-on option value increases with both the volatility of operating revenue and the volatility of operating costs. The former effect may be inferred from the real option's protection against downside risks. The latter effect (which has previously been ignored because the operating costs are usually assumed to be fixed) arises because the option allows an investor the opportunity to profit at lower operating costs when the volatility of operating costs increases.

Finally, we show that when an individual investor disagrees with the market about the premium associated with a risk factor involved in the project, hedging against this risk should reduce the add-on option value. By contrast with the incomplete market models which assume perfect hedging of all hedgeable project risks, the hedging strategy is not central to our arguments. Instead, we focus on the difference between (a) individual perception of risk premium associated with an uncertainty affecting the project value and (b) the risk premium associated with representative investor. This is not to say that hedging is irrelevant for the add-on value. Indeed, the hedging effect promotes the importance of investigating the idiosyncratic risk effect on the add-on value. In reality, it is likely that the investor will agree with the market more on the systematic risk premia than on idiosyncratic risk premia. Hence, it is more often the case that add-on value arises from bearing idiosyncratic risks than systematic risks.

In the following: Section 2 introduces our risk-adjusted valuation model; Section 2.3 applies the model to a case study on an option to acquire a gold deposit; Section 3 investigates the effects that different parameters have on the add-on value, formulating a number of hypotheses which are explored by simulation. The section also examines the effect of hedging on the add-on value and Section 4 concludes.

2 The Model

Consider an imperfect market with no arbitrage. Suppose a risk averse investor is interested in acquiring an irreversible investment project in any year before it expires at some finite time T . Denote the equilibrium project value by s_t^{eq} , $0 \leq t \leq T$. This is the market clearing price for the project which reflects the financing efficiency and risk preferences of the market representative investor (also see Chapter 13 in [Brealey and Myers, 2000](#)). As implied by [Myers \(1977\)](#); [Quigg \(1993\)](#); [Grenadier \(2002\)](#); [Reyck, Degraeve, and Vanderborre \(2008\)](#), s_t^{eq} can hence be interpreted as the expected profit of the individual investor paid ahead of his investment if he shares the same financing efficiency, risk preferences and subsequently, the same optimal strategy as the market representative investor. The project operation generates a stream of revenue, x_{1t} , associated with a series of costs, x_{2t} . We adopt the conventional assumption that their evolutions under the physical measure, \mathcal{P} , can be described by two correlated geometric Brownian motions (GBM):

$$\Delta \ln(x_{it}) = \ln\left(\frac{x_{i,t+\Delta t}}{x_{it}}\right) = \left(\mu_{it} - 0.5\sigma_{it}^2\right)\Delta t + \sigma_{it}\sqrt{\Delta t}\mathcal{Z}_{it}^{\mathcal{P}}, \quad (1a)$$

$$\mathcal{Z}_{it} = \rho_{it}\mathcal{B}_0 + \sqrt{1 - \rho_{it}^2}\mathcal{B}_i, \quad i = 1, 2, \quad 0 \leq t \leq T, \quad (1b)$$

where: μ_{it} and σ_{it} denote the expected growth rate and the volatility of the cash flow stream x_{it} ; \mathcal{B}_j , $j = 0, 1, 2$, are three orthogonal Brownian motions; and ρ_{it} is the correlation between \mathcal{Z}_{it} and \mathcal{B}_0 .

Rather than assume a process for the project value evolution directly (eg. in [Henderson, 2007](#); [Grasselli, 2011](#)), we prefer to model the project cash flows and then the project value accordingly. While some complete market models (such as in [Copeland and Antikarov, 2001](#)) are also developed under cash flow assumptions, our model is more general because we are not limited to the case of a complete market. Moreover, modelling the project's operating revenues and costs as two separate GBMs is much more intuitive than setting the operating revenues net of costs (*i.e.* the profits) as a GBM. The use of both revenue and cost functions is consistent with

a potential for negative profit. Further note that, in contrast to several important works (*eg.* see [Myers, 1977](#); [Henderson, 2007](#)) and their applications (*eg.* see [Insley, 2002](#); [Grasselli, 2011](#)) which assume fixed operating costs, it is more realistic to assume stochastic operating costs which are correlated with the operating revenues. This is more likely to be the case in practice where project operations are likely to consume energy commodities with volatile market prices (also see [Slade, 2001](#)).

2.1 Project Value and Expected Return

This section revisits the DCF framework in which we calculate the project value in line with traditional investment valuation. There can be two values for the same project: that estimated by the market representative investor, *i.e.* the market clearing price s_t^{eq} , and that by the individual investor, s_t . Mathematically, the expected return on the project forecasted by the market, r_t^{eq} (hereafter the “implied return”), is implied by the following relationship:

$$s_t^{eq} = \sum_{\tau=t+1}^T \mathbb{E}_t^{\mathcal{P}} [x_{3\tau}] \exp \left(- \sum_{k=t+1}^{\tau} r_k^{eq} \right), \quad (2)$$

where x_{3t} denotes the operating profit at time t , *i.e.* $x_{3,t} = x_{1t} - x_{2t}$, and $\mathbb{E}_t^{\mathcal{P}}[\cdot]$ denotes the time t expectation under the \mathcal{P} measure. Hence, in contrast to classic models such as [Copeland and Antikarov \(2001\)](#), [Henderson \(2007\)](#) and others, the market clearing price is stochastic.

While s_t^{eq} should reflect the real option value perceived by the market (as should r_t^{eq}), an individual investor may obtain a very different estimate for the project value, s_t . If most investors (and hence, the market) are aware of the investment opportunity, and accordingly expect to generate a profit additional to the market clearing price, then this price should simply increase by one amount of the expected profit. A similar argument is applicable for the implied return. Correspondingly, the pay-off expected by the individual investor to acquire the project at time t , *i.e.* $\text{NPV}_t = s_t - s_t^{eq}$ can differ from zero. We follow the convention of referring to NPV_0 as the project NPV. Usually, a non-zero NPV arises because the expected cost of capital for the project estimated by the individual investor, r_t , differs from the implied return r_t^{eq} . Similar to

(2), r_t and s_t are related in the following manner,

$$s_t = \sum_{\tau=t+1}^T \mathbb{E}_t^{\mathcal{P}} [x_{3\tau}] \exp \left(- \sum_{k=t+1}^{\tau} r_k \right). \quad (3)$$

We then refer to r_t as the “required project return”, which will be specific to the individual investor.

Clearly, a project NPV can differ between individual investors. A non-zero NPV arises because the individual’s beliefs diverge from those of the market representative investor. These beliefs pertain to the project value (or equivalently, to the project’s expected return) and they reflect differences in financing efficiency and risk preferences resulting from their different knowledge, information, operating efficiency and perceptions. A lower (higher) required return relative to the implied rate leads to a positive (negative) NPV_t , and this may be considered as an indication of the individual investor being more risk loving (averse) or financially efficient (inefficient) when compared to the market representative investor. It also follows that a non-zero NPV cannot be obtained from a perfectly hedged portfolio comprising the project and other assets.

2.2 Option to Acquire the Project

This section concerns an option of the individual investor to acquire the project in any year before the project expires. The market representative investor can perceive some real option value and equivalently, determine an optimal investment policy associated with the project. As argued previously, this value is part of the equilibrium project value. But we focus on the individual investor’s flexibility to execute his own optimised investment strategy in response to the market belief. It follows our previous discussion about the equilibrium project value that only by using such a strategy can the investor obtain the maximum non-negative profit from the project, net of all costs incurred by the strategy. As stated earlier, we refer to this net profit as the add-on value of the real option.

We only model this add-on value and ignore the real option value perceived by the market, for the following reasons: first, modelling the add-on value is equivalent to finding the optimal

investment policy for individual investors; second, it is usually too complicated to determine the option value perceived by the market, given that the equilibrium value may include the market values of several interacting real options;² third, it may not be of interest to the individual investor to compute the market value of the option since he and the market representative investor can have very different financing efficiency and risk preferences. Even if the investor follows the market in every respect, he would only generate a zero NPV. Instead, he would choose to exercise the real option based on his individual interests, and to obtain a non-negative add-on value accordingly. More specifically, with the option to acquire the project, the investor can either: (i) invest immediately if the project NPV is positive, (ii) decline, or (ii) defer the investment if he can obtain a higher expected profit than the project NPV. Choosing between (i) and (ii) follows the zero NPV rule and generates either zero or a positive project NPV.

Given the value of the expected pay-off to the individual investor on acquiring the project at time t , NPV_t , the add-on value of the option (*i.e.* the value associated with the opportunity of investing now or in the future net of all cost), Φ_0 , is given by:

$$\Phi_0 = \sup_{0 \leq t \leq T} \left\{ \max \left(NPV_t^{\mathcal{Q}}, 0 \right) \right\}, \quad (4)$$

where \mathcal{Q} denotes a risk-adjusted measure (not necessarily a martingale measure) that is equivalent to the \mathcal{P} measure. Under this \mathcal{Q} measure, all values under the \mathcal{P} measure are presented in their PV terms according to the individual investor's unique financing efficiency and risk preferences across all assets. This construction allows the option pay-off NPV_t from different points in time $0 \leq t \leq T$ to be comparable with each other without using any restrictive assumption such as perfect replicability for NPV_t or involving a specific utility function. The associated value of delaying the investment (*i.e.* the value associated with the opportunity of investing in the future), φ_0 , which is captured by our model while ignored by the DCF approach, hence takes the following form:

$$\varphi_0 = \Phi_0 - \max \left(NPV_0, 0 \right).$$

Also see [Copeland and Antikarov \(2001\)](#) for the same calculation of the value of delaying the

²Refer to [Damodaran \(2000\)](#) for a similar argument and [Trigeorgis \(1993\)](#) for investigations on the interactions between real options attached to the same project.

investment.

The set-up of target function (4) further contributes to the literature by providing the following intuitions. First, by definition, $\text{NPV}_t^Q = s_t^Q - s_t^{eq,Q}$. Indeed, the option strike is the market clearing price s_t^{eq} which reflects the stochastic views of the market about the project value. Hence, (4) displays an important intuition that Φ_0 is always zero if NPV_t is zero, and this happens when the individual investor sharing the same financing efficiency and risk preferences as the market representative investor. The classic models (such as Copeland and Antikarov, 2001; Henderson, 2007) fail to draw the same conclusion because they ignore the uncertainty in the project's acquisition price (to which the subjective project value converges). Mathematically, with a fixed strike K (i.e. a constant acquisition price for the project), $\text{NPV}_t^Q = s_t^Q - K$ could not always be 0 for $0 \leq t \leq T$ when s_t^Q is stochastic and subsequently, Φ_0 could not be zero everywhere. In other words, the add-on option value generated by classic models (which set a fixed strike) can only be 0 if the individual investor believes that the project value is constant and equal to the strike. However, this is illogical given that the investor is aware of the uncertainty involved in the project.

Second, given that the option pay-off NPV_t is subjective, the add-on value Φ_0 is also subjective. One can consider Φ_0 as a reward to the individual investor over and above the market clearing price of the project; it can only be realised if the investor exercises the real option based on his own optimal strategy in response to the market belief.

Third, this also forms the basis of our theoretical critique of complete market models which claim to capture a real option value as the market price of the real option if it were tradable by adopting the risk-neutral valuation techniques. Copeland and Antikarov (2001)'s justification of using such techniques leads to extracting the real option value incorporated in the equilibrium project value s_t^{eq} instead of modelling Φ_0 which is additional to s_t^{eq} .

Note that a zero value for Φ_0 does not necessarily imply that the flexibility of the individual investor to make his own investment strategy in response to the market belief is valueless. Instead, following Myers (1977); Copeland and Antikarov (2001); Henderson (2007); Grasselli (2011) and others, it denotes a zero expected pay-off from the project given by the individual

investor who utilises his flexibility optimally. For instance, suppose the project NPV is negative. If $\Phi_0 = 0$, the flexibility has in effect, generated a delay value of $\Phi_0 = -\text{NPV}_0$.

Now, to obtain Φ_0 and φ_0 using (4), we compute $\text{NPV}_t^{\mathcal{Q}}$ as follows,

$$\text{NPV}_t^{\mathcal{Q}} = \text{NPV}_t \exp \left(- \sum_{\tau=1}^t r_{\tau} \right), \quad \text{NPV}_0^{\mathcal{Q}} = \text{NPV}_0. \quad (5)$$

That is, we first calculate the PV of the subjective project value by discounting the project value at r_t , the discount rate for the operating profit stream, specified by the individual investor for computing the subjective project value (3), *i.e.*

$$s_t^{\mathcal{Q}} = \sum_{\tau=t+1}^T \mathbb{E}_t^{\mathcal{P}} [x_{3,\tau}] \exp \left(- \sum_{k=1}^{\tau} r_k \right) = s_t \exp \left(- \sum_{k=1}^t r_k \right).$$

Second, we argue that the individual investor would discount market clearing prices at different times using the same rate r_t :

$$s_t^{eq,\mathcal{Q}} = s_t^{eq} \exp \left(- \sum_{k=1}^t r_k \right),$$

because for the individual investor, it is only a different estimate of the same project as opposed to the subjective value, *i.e.*, they share the same sources of uncertainty. Finally, (5) follows from $\text{NPV}_t^{\mathcal{Q}} = s_t^{\mathcal{Q}} - s_t^{eq,\mathcal{Q}}$.

A technical advantage of our model is that: we discount the expected pay-off from the underlying project before computing the option value instead of discounting the option pay-off directly. In this way, we overcome the alleged difficulty of finding a risk-adjusted discount rate appropriate for the option pay-off (*eg.* see [Dixit and Pindyck, 1994](#)). Our set up is consistent with utility-based real option models such as [Henderson \(2007\)](#); [Grasselli \(2011\)](#). In their framework, given the assumed utility function of the investor, the certainty equivalent of the future profit is equivalent to the discounted expected profit in our model. Also see [Levy and Markowitz \(1979\)](#); [Markowitz \(2014\)](#) for further details of such equivalence. In comparison to utility-based models, however, our model relies on risk-adjusted discounting of expected future profit. This aids consistency between the DCF valuation and a real option valuation especially

in terms of the financing efficiency and risk preferences of individual investors. Importantly, our model requires exactly the same assumptions and parameters as those needed in the DCF framework. In this respect, our approach is more practical than both complete market models and utility-based models.

2.3 Example Application

To illustrate the practicality of our approach, we value an option to acquire a gold deposit. Such options are crucial to mineral producers such as Barrick Gold.³ As the largest gold producer in the world, it has acquired numerous high-quality mines during the past twenty years. In 1994 it took over Lac Mineral Ltd and hence acquired several large deposits including the El Indio Belt and the Veladero Project. In 1996 it acquired Aneguiipa Resources Ltd. and became the sole owner of the Pierina deposit. In 2000, 2001, 2006, 2007, 2008 and 2010, Barrick Gold acquired at least 16 high-quality mines, especially in 2006 when it acquired Placer Dome Inc., adding a dozen new deposits into its global portfolio. By 2009, Barrick Gold had grown to hold the industry's largest reserves. Barrick Gold has also announced, many times, that its only operating target is to operate high-quality mines. Interestingly, in 2008 it eliminated all hedges of the gold price.

We value the option of a hypothetical gold mining firm, AWC, to acquire a hypothetical high-quality deposit, Velaguna, whose production is expected to be 0.40 million ounces of gold every year from 2014 to 2018.⁴ From 2019 on, it is assumed that it will no longer be economical to continue production.⁵ The total cost of the deposit operation in 2013 was \$450 million, half of it spent on buying fuel oil, the cost of this being the main component of uncertainty in the cost. If AWC were to acquire the mine at the end of 2013, the acquisition price would be \$240.16

³However, the academic literature generally treats the option to open/close mineral operations as the more important. Options to suspend/resume production are also of second order (see [Slade, 2001](#)). To some extent, an option to acquire a deposit is very similar to an option to open a deposit operation in terms of option pay-off. In fact, it is very often much simpler as it does not involve such complex cost structure.

⁴The common measures for the quality of a gold deposit include the size of gold reserves and the average grade. Both measures determine the level of costs involved in deposit operation.

⁵The benchmark for deciding if it is economical to continue a gold deposit operation is the cut-off grade of gold ore which is a function of the average operating revenues and costs. It is only economical to continue the operation if the processing grade of the gold ore that is mined is above the cut-off grade.

million.

Now, while the annual production is a constant, the expected growth rate and the volatility of the operating revenue, *i.e.* μ_{1t} and σ_{1t} , would be equal to the annualised average weekly return and the volatility of returns of the price of gold per ounce, respectively, assuming 52 weeks in a year. To estimate these parameters weekly gold spot prices from 2009 to 2013 were acquired from Bloomberg, as were all other market data used in this case study. The spot price of gold per ounce was \$1213.27, at the end of 2013. Hence, the initial operating revenue $x_{1,0} = 4 \times 10^5 \times 1213.27 = \4.85×10^8 , slightly higher than the initial operating cost $x_{2,0} = \$4.50 \times 10^8$. The annualised average weekly return and the annualised volatility of weekly returns on the gold spot is 15.99% and 17.56%, respectively. Note that, the average return of the gold spot price and its variance are computed as the average return and variance of returns of five non-overlapping annual windows of weekly data. Such estimation is adopted for all average returns and variances in this case study.

To estimate the growth rate and the volatility of the operating costs, μ_2 and σ_2 , we use weekly returns on the Latin American Vasconia crude oil index between 2009 and 2013; the annualised average weekly return and its volatility are 30.88% and 34.24%, respectively. Hence $\mu_2 = 0.5 \times 30.88\% = 15.44\%$, and $\sigma_2 = \sqrt{0.5} \times 34.24\% = 24.21\%$, following the assumption that half of the cost is spent on oil and the other half are constant. The correlation between the weekly returns on the gold spot and the crude oil index 28.14%.

Furthermore, using definition (2), the implied rate of return $r^{eq} = 12\%$, *i.e.* it satisfies the following equation given that the acquisition price stands at $\$2.4016 \times 10^8$ and the initial revenue (cost) is $\$4.85 \times 10^8$ ($\$4.50 \times 10^8$) million, growing at 15.99% (15.44%) per year:

$$2.4016 \times 10^8 = \sum_{\tau=1}^5 \left(4.85 \times 10^8 \times \exp(0.1599 \tau) - 4.50 \times 10^8 \times \exp(0.1544 \tau) \right) \exp(-r^{eq} \tau).$$

In comparison to the implied return, this case study assumes that AWC expects a 10% cost of capital associated with the Velaguna program, slightly lower than the market offer.⁶

⁶It is common practice for gold producers to estimate an expected cost of capital associated with an on-going or potential deposit operation, based on statistics from other comparable operating mines (see Brealey and Myers, 2000, for further justifications of using comparable portfolios). We suppose Velaguna, as a high-quality mine, is

The inputs of the model parameters are summarised as follows:

$$\begin{aligned}
 T = 5, \quad x_{1,0} = 4.85 \times 10^8, \quad x_{2,0} = 4.50 \times 10^8, \quad r = 10\%, \quad r^{eq} = 12\%, \\
 \mu_1 = 15.99\%, \quad \sigma_1 = 17.56\%, \quad \mu_2 = 15.44\%, \quad \sigma_2 = 24.21\%, \quad \rho = 28.14\%.
 \end{aligned}
 \tag{6}$$

where ρ denotes the correlation between the operating revenue and costs.⁷ In comparison to the acquisition price of Velaguna, 240.16 \$m, its current subjective value (3) for AWC stands at 256.10 \$m. Therefore, the NPV of the project, $NPV_0 = 256.10 - 240.16 = 15.94$ \$m, indicating that AWC is comparatively more efficient or less risk averse in comparison to the market in operating the deposit.

We now employ the least-squares Monte Carlo simulation technique developed by [Longstaff and Schwartz \(2001\)](#) to approximate the add-on option value Φ_0 .⁸ We use the above values for model parameters to simulate a 10^5 paths for the values of the operating cash flows, and thus obtain the add-on value of the option to acquire Velaguna as $\Phi_0 = 19.57$ \$m. This is to say, ABY expects to generate 19.57 \$m abnormal profit from operating Velaguna if ABY takes over it at some time within the five-year time horizon providing ABY's financing ability and risk preferences. [Figure 1](#) depicts the upper and lower early exercise boundaries of ABY's option to invest in Velaguna. The red and blue lines depicts the option pay-off at time t , NPV_t^Q at which ABY would be indifferent between exercising the option immediately and delaying it. When NPV_t^Q lies between the two lines, ABY would prefer to exercise the option immediately than delaying it. Given the project $NPV = 15.94$ \$m, it is better for ABY to postpone the decision to invest at time 0 and from this, ABY can generate an abnormal profit up to $\varphi_0 = 19.57 - \max\{15.94, 0\} = 3.63$ \$m.

comparable to existing high-quality mines in terms of their operational characteristics including average ore grade, size and on-field facility capacities.

⁷Note that, instead of estimating ρ_1 and ρ_2 for operating cash flows (1a), we directly estimate the product of these two correlation coefficients, *i.e.* ρ , which is enough for applying our model to this case study.

⁸Note that, for real options with complicated pay-offs, one can alternatively simulate the add-on value using the binomial tree option pricing methods or finite difference methods introduced by [Schwartz \(1977\)](#); to improve the computational speed for the simulation, one can employ the quadrature methods introduced by [Andricopoulos, Duck, Newton, and Widdicks \(2003, 2007\)](#); [Chen, Harkonen, and Newton \(2014\)](#).

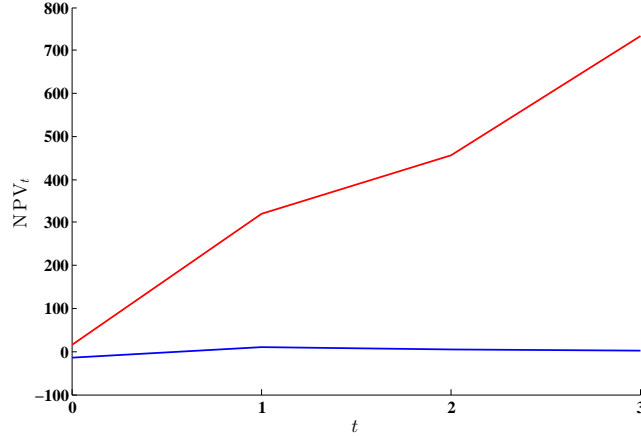


Figure 1: Early exercise boundaries of ABY's option to invest in Velaguna between now and year 5. The red and blue lines depicts the upper and the lower boundaries, respectively. Parameter inputs are provided by (6).

3 Simulation Results

In the following we investigate four hypotheses about the effect of different parameters on the add-on option value.

All simulations are based on the following base parameter input values:

$$T = 5, \quad x_{i,0} = 5, \quad \mu_i = 40\%, \quad \sigma_i = 30\%, \quad \rho_i = 0.5, \quad r^{eq} = 20\%, \quad r = 18\%. \quad (7)$$

(7) sets the distributions of the operating revenue and cost to be the same so that effect of changes in the operating revenue and cost on the add-on option value are of comparable magnitude. Also, the expected growth rates for the operating cash flows are set high in (7) so that the expected cash flows increase faster over time and subsequently, the discount rate effect on the add-on value becomes more obvious for observation than if the expected cash flow growth rates were set low. Various inputs are employed for the initial cash flows, the cash flow volatilities and correlations, and the expected project returns. Qu our simulation results concerning hypotheses $H_1 \sim H_4$. With the inputs in (7) the project NPV is zero (*i.e.* the option to acquire the project is an at-the-money option) and so, under DCF, investors would be indifferent between investing

- H_1 : The add-on real option value increases with the volatility of operating revenues and with the volatility of operating costs, and decreases with the correlation between costs and revenues.
- H_2 : The add-on real option value generally increases with the difference between the required and implied project return, indicating that this value arises from the difference between the individual investor and the market representative investor in terms of financing efficiency and risk preferences. In the case when the required return is already extremely high in comparison to a given implied return, the add-on real option value might decrease with further increases in the required return.
- H_3 : The relationship between the idiosyncratic risks and add-on real option value is not monotonic: it is generally positive but it can be negative if the individual investor is extremely inefficient or risk averse.
- H_4 : The effect of hedging a source of risk involved in the project depends on the agreement between the individual investor and the market about the premium for this risk: if they agree, then hedging this risk should not affect the add-on option value attached to the project; otherwise, such hedging should reduce the option's add-on value.

now or not. Assuming a positive NPV does not add to our analysis; it just gets carried into Φ_0 . Hence, when parameters are at the base value, the add-on value of the option $\Phi_0 = 0.4293$, the same as the value of delaying the investment, φ_0 .

Considering first H_1 : the left panel in Figure 2 depicts the add-on value of the real option for a range of revenue volatilities σ_1 . For this we adopt the most popular assumption in the literature, setting the operating costs x_{2t} as constants (also see [Howell, Stark, Newton, Paxson, Cavus, Pereira, and Patel, 2001](#)). This panel shows a monotonic and positive relationship between the add-on option value and the volatility σ_1 which is consistent with the general belief in the literature that the more volatile is the project value, the more valuable is the option to invest

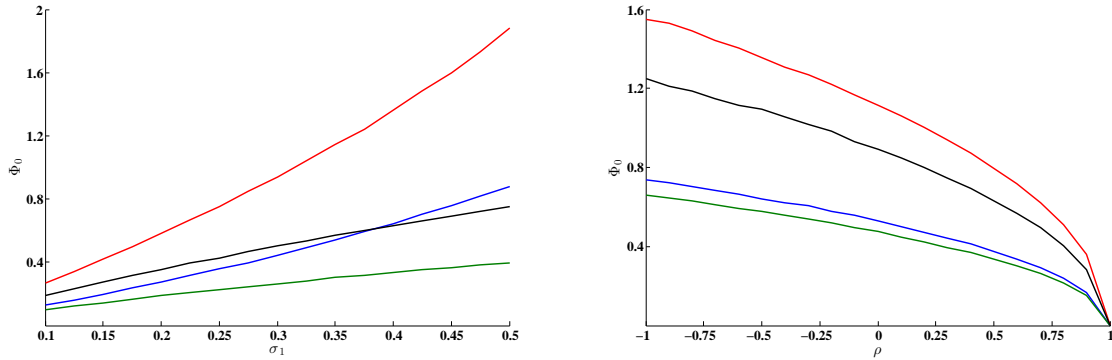


Figure 2: The left panel depicts the add-on value of the real option for a range of revenue volatilities σ_1 , with the cost volatility $\sigma_2 = 0$. The right panel depicts the add-on value of the real option for a range of correlations between revenues and costs ρ . For both panels, the red, blue, green and black lines correspond to the values of the required return $r = 0.16, 0.18, 0.22, 0.24$, respectively.

in it (eg. see Copeland and Antikarov, 2001; Henderson, 2007).

Note that, with the implied return r^{eq} set equal to 0.20, the red (black) line corresponding to the value of the required return $r = 0.16$ (0.24) is higher than the blue (green) line which corresponds to the values $r = 0.18$ (0.22). This indicates an increase in the option value resulting from the absolute difference between r^{eq} and r , i.e. the expected returns perceived by the market and the individual investor, respectively. See the next section for further discussions of the effects of required returns and risk aversion of the investor on the add-on value of the real option.

The right panel of Figure 2 depicts a monotonic and negative relationship between the add-on option value and the correlation between the operating revenue and cost streams ρ . It is intuitive given that positively correlated revenues and costs would naturally offset each other's risks. The increase in the value of ρ therefore reduces the overall risks of the project and subsequently, lowers the add-on value. The extreme case is when ρ increases to 1. The project then becomes risk-free as the revenues are perfectly hedged by the costs given our inputs. The add-on option value hence becomes 0. Moreover, consistent with the previous figure, the red line with $r = 0.16$ is higher than the blue line with $r = 0.18$ and similarly, the black line with $r = 0.24$ is higher than the green line with $r = 0.22$. Both indicate a reduction of the add-on value following the convergence of r to r^{eq} which is set as 0.20.

Our preliminary analysis has confirmed the positive relationship between the add-on option value and the volatility of the operating revenue, which is usually justified by the view that the real option protects the individual investor from downside risks, *i.e.* the individual investor can choose not to take on the project if he cannot generate a positive profit NPV_t . In addition, Figure 3 depicts the add-on value for different operating cash flow volatilities, σ_1 and σ_2 . It can be seen that the relationship between the add-on value and the volatility of the operating costs is also monotonic and positive. This result has been ignored by the literature in which the operating

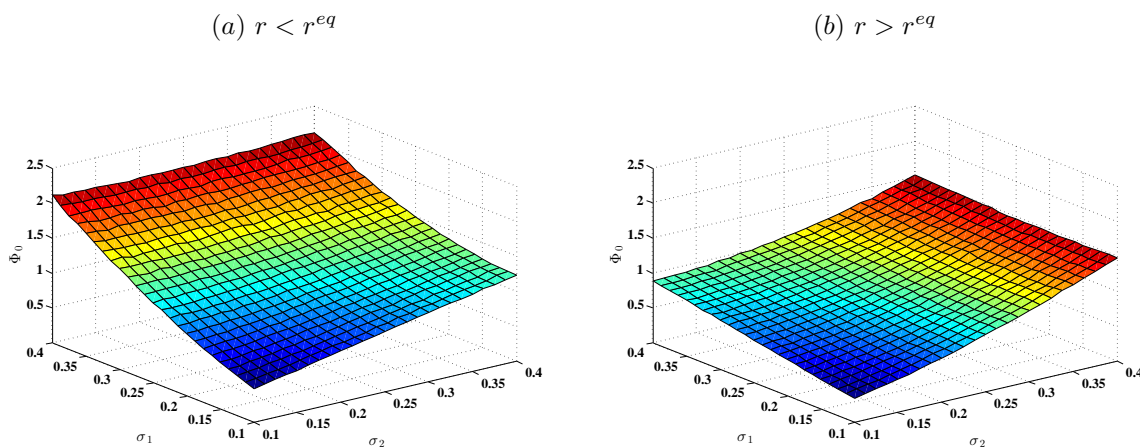


Figure 3: The add-on option value for a range of the operating cash flow volatilities $\sigma_1, \sigma_2 \in [0.1, 0.4]$. For the left (right) panel, $r = 0.14$ (0.26), 6% lower (higher) than the implied rate of return $r = 0.20$.

costs are usually assumed as constant, although Slade (2001) has argued that stochastic costs are very common in practice.

Obviously, such positive effect of the volatility of operating costs cannot stem from the real option being a protection from downside risks. In fact, the more uncertain are the operating costs, the less valuable should be the protection. We argue that this effect arises from an additional function of the option to invest, that is, it allows the individual investor to profit when the costs are low. More specifically, when the volatility of the operating costs increases, the investor has more chance to obtain any given revenue while the costs turn out low and subsequently, the expected pay-off of the option increases as does the add-on value.

Given that the revenue and the cost volatility effects should offset each other, Figure 4 shows

that the relative effect depends on the relationship between the required and the implied returns. It depicts the add-on option value for a range of volatilities of operating revenues or costs, σ_1

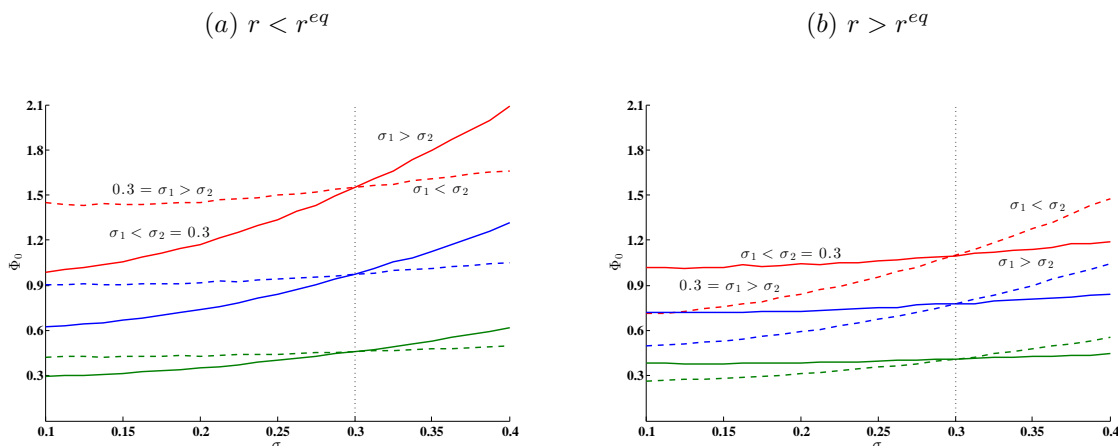


Figure 4: The option add-on value as the volatility of operating revenues or costs, σ_1 or σ_2 varies between 0.10 and 0.35. All solid lines are with a varying σ_1 while $\sigma_2 = 0.3$, and all dashed lines are a varying σ_2 while $\sigma_1 = 0.3$. In the left panel, the red, blue and green lines take $r = 0.14, 0.16$ and 0.18 , respectively, whereas in the right panel, the red, blue and green lines take $r = 0.26, 0.24$ and 0.22 , respectively.

or σ_2 . The left panel takes the required return lower than the implied return which indicates the individual investor being more efficient financially or less risk averse relative to the market. Between each pair of lines with the same colour, the higher line is always with the revenue volatility higher than the cost volatility. This implies a stronger effect of the revenue volatility on the add-on value as opposed to the effect of the cost volatility. On the other hand, when the required return exceeds the implied return as in the right panel, such comparative relationship is reversed.⁹

We have involved both the implied and required project returns throughout the real option valuation. These two rates have a critical impact on the investors' optimal investment strategy and equivalently, the add-on option value (given that they are strong indicators of the investors'

⁹Moreover, all lines in the left panel of Figure 4 are higher than lines in the right panel with the same types and colours. This is intuitive in that the flexibility of the individual investor to take his own strategy in response to the project risks would become more valuable when he becomes more efficient or less risk averse relative to the market representative investor. Figure 4 also shows that the solid (dotted) lines in the left (right) panel are steeper than the lines with the same types and colours in the right (left) panel. This is simply because the solid lines in the left panel correspond to varying revenue volatilities which create the dominating effect on the add-on value, whereas the dotted lines in the right panel correspond to varying cost volatilities whose effect is dominated.

financing efficiency and risk preferences). As argued in Section 2, a lower (higher) required return relative to the implied return indicates that the individual investor is more risk loving or financially efficient (inefficient) than the market representative investor.¹⁰

Next we consider H_2 . To this end, Figure 5 depicts the add-on option value as the project NPV varies between -9 and 6 . Consistent with our previous discussion, both panels show an

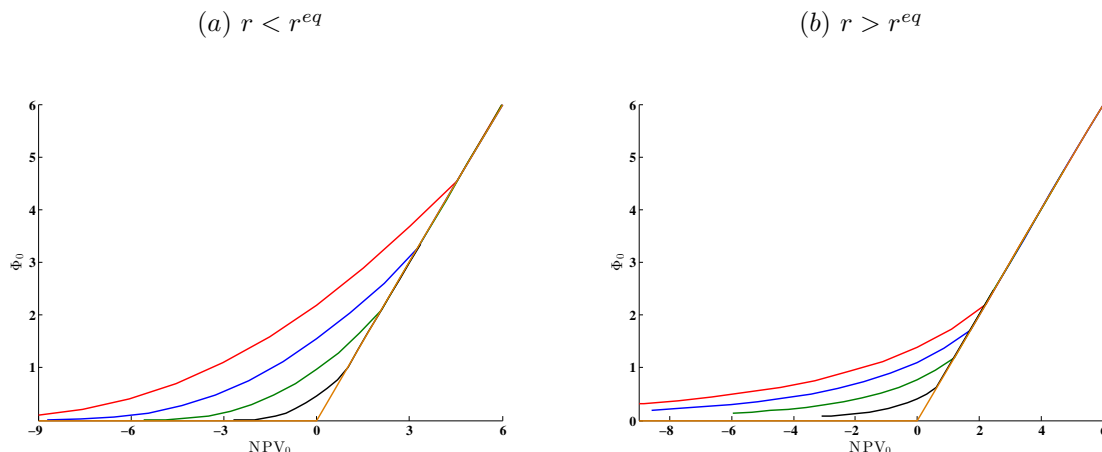


Figure 5: The real option value for a range of the project NPV. For both panel, $x_{1,0} \in [1, 10]$. The red, blue, green and black lines in the left panel take the values of the required return $r = 0.12, 0.14, 0.16, 0.18$, respectively, and in the right panel, they take $r = 0.28, 0.26, 0.24, 0.22$, respectively. The orange lines in both panels depict the option pay-off at time 0.

increase in the add-on value of the real option with the absolute difference between the implied and required returns: the four lines depicting the add-on value in each panel correspond to 0.08, 0.06, 0.04 and 0.02 absolute difference from highest to lowest. This illustrates that the option value generally arises from the divergence in financing efficiency and risk preferences between the market and the individual investor.

A more general description of the impact of the implied and required returns on the add-on value is presented by Figure 6. For this, the project NPV is set as zero which holds for all the following results. The right panel takes four slices of the add-on value surface depicted in the left panel at the implied return $r = 0.10, 0.15, 0.2$ and 0.3 , respectively. Results in both

¹⁰See Levy and Markowitz (1979); Markowitz (2014) for justifications for using the expected return and variance of returns to represent the investor's risk aversion, disregarding higher moments of the return distribution. For each line or surface presented in this section, the cash flow volatilities are fixed and hence, the risk aversion of the investor becomes a linear function of the expected return.

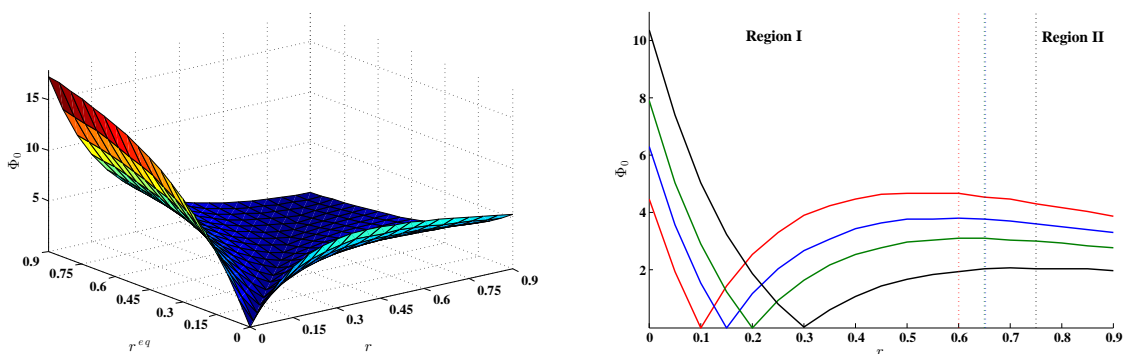


Figure 6: The add-on option value corresponding to different values of the implied and the required returns. The left panel takes $r^{eq}, r \in [0, 0.9]$. The red, blue, green and black solid lines in the right panel are associated with $r^{eq} = 0.1, 0.15, 0.2,$ and $0.3,$ respectively. Each dotted lines crosses the solid line with the same colour when it starts to fall.

panels generally confirm our previous finding that the add-on value increases with the absolute difference between the implied and required return (for instance, see region I in the right panel). However, this relationship can be reversed when the required return is much higher than the implied return; for instance, see region II in the right panel with the required return r at least over 55%, more than twice as much as the implied return. This is to say, when the individual investor is already extremely inefficient or risk averse in comparison to the market, the value of the individual investor's flexibility of making his own investment strategy can fall in value if he continue increases his required return on the project.

Particularly, the seemingly counter-intuitive increase in the add-on value with the required return (when this return is slightly higher than the implied return), results from the volatility effect on the add-on value. The higher the absolute difference between the required and implied return, the larger is the actual dispersion of the expected pay-off from the project NPV_t which then gives rise to the option value. On the other hand, when the required return becomes extremely high, the chance of obtaining a positive NPV_t reduces disproportionately with the increase in the required return and hence, the option value falls.

Our results here contradict some of the conclusions about the effect of the individual investor's risk aversion on the add-on value in a utility-based framework. In our set-up, this effect is not necessarily monotonic and negative, as in [Henderson \(2007\)](#). The contradiction mainly

stems from her unrealistic assumptions of an infinite life and a fixed strike for the real option.

For H_3 , we suppose that the source of risk, \mathcal{B}_0 , shared by the operating cash flows (1), is a systematic risk factor whereas the two other risk factors, \mathcal{B}_1 and \mathcal{B}_2 , are idiosyncratic. We denote by θ_j^{eq} , $j = 0, 1, 2$, the market premia for all three source of risks, \mathcal{B}_j , respectively, and by θ_j the premia required by the individual investor. We further assume no hedging and that the individual investor agrees with the market on the systematic risk premium: $\theta_0^{eq} = \theta_0 = 2/3$. Moreover, we set the market premia for the idiosyncratic risk factors, $\theta^{eq} := \theta_1^{eq} = \theta_2^{eq} = -0.5, 0.5$ and denote the idiosyncratic volatility and its associated premium required by the investor by $\sigma^{ids} := \sigma\sqrt{1-\rho^2}$ where $\sigma := \sigma_1 = \sigma_2$ and $\rho := \rho_1 = \rho_2$, and $\theta^{ids} := \theta_1 = \theta_2$, respectively. The implied and required returns, r^{eq} and r , can be computed given the risk premia:

$$\begin{cases} r^{eq} = \left[\theta_0^{eq} \rho + \theta^{eq} \sqrt{1 - \rho^2} \right] \sigma = \frac{\theta_0^{eq} \sigma^{ids}}{\sqrt{1 - \rho^2}} + \theta^{eq} \sigma^{ids}, \\ r = \left[\theta_0 \rho + \theta^{ids} \sqrt{1 - \rho^2} \right] \sigma = \frac{\theta_0 \sigma^{ids}}{\sqrt{1 - \rho^2}} + \theta^{ids} \sigma^{ids}, \end{cases} \quad (8)$$

respectively.

Figure 7 depicts the add-on value of the option to invest for different values for σ^{ids} and θ^{ids} . This figure illustrates the non-monotonic relationship between the add-on value and the

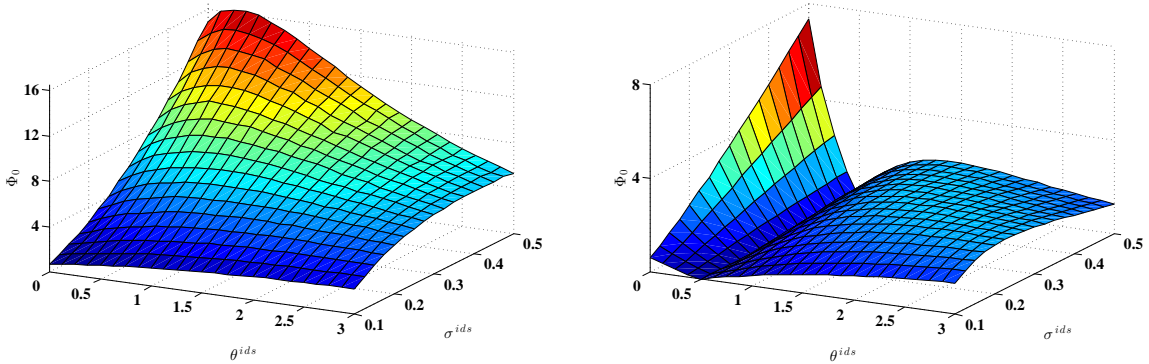


Figure 7: The add-on option value for a range of idiosyncratic risk $\sigma^{ids} := \sigma\sqrt{1-\rho^2}$ and required premium $\theta^{ids} := \theta_i$, $i = 1, 2$. For both panels, $\theta_0^{eq} = \theta_0 = 2/3$. The market premia for the idiosyncratic risk factors, $\theta^{eq} := \theta_i^{eq} = -0.5$ for the left panel whereas for the right panel, $\theta^{eq} = 0.5$.

idiosyncratic risk. In particular, this relationship is generally positive, consistent with our previous investigations on the volatility effect. Such relationship, however, can be reversed when the individual investor becomes extremely risk averse for the risk factor. For instance, see Figure 8 which takes four slices of the surface in the right panel at $\theta^{ids} = 1, 1.5, 2, 3$. Three out of

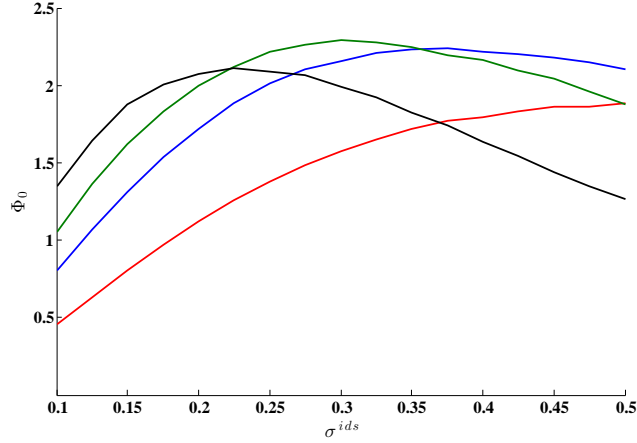


Figure 8: The add-on option value for a range of idiosyncratic risk σ^{ids} . Four lines all take $\theta_0^{eq} = \theta_0 = 2/3$ and $\theta^{eq} = 0.5$. The red, blue, green and black lines take $\theta^{ids} = 1, 1.5, 2, 3$, respectively.

four lines, show a decay of the add-on value when the idiosyncratic risk exceeds approximately 20 ~ 40%. Specially, the left panel results in Figure 7 accord with [Ang, Hodrick, Xing, and Zhang \(2006\)](#) and sets a negative market premia for two idiosyncratic risk factors. The figure shows that even though the negative risk premia offered by the market would discourage the investor from taking on the project, the investor can still prefer to bear the risk and generate a positive net profit from the project by investing with optimal timing according to his financing efficiency and risk preferences.

Finally, we remark that [Henderson \(2007\)](#) identifies a parameter region defined by

$$\zeta := 1 - \frac{2(r^{eq}/\sigma - \theta_0^{eq}\rho)}{\sigma} \in [0, 1]$$

in which the add-on option value can be finite, indicating that the investor may exercise the real option before it expires. But when ζ falls below 0 the add-on value of the option becomes infinitely large, *i.e.* the real option becomes too dear to be exercised. Our results agree with this

property when $\zeta > 0$ but not when $\zeta < 0$. In the latter case we can generate a finite add-on option value, implying the possibility that the investor exercises the option before its expiry. To see this, we use (8) to rewrite

$$\zeta = 1 - \frac{2 \theta^{eq} \sqrt{1 - \rho^2}}{\sigma},$$

so the special parameter region (7) is effectively $0 \leq \theta^{eq} \leq 0.5 \sigma (1 - \rho^2)^{-0.5}$. Table 1 depicts the add-on option value for a wide range of values for ζ where we adopt Henderson (2007)'s parameter inputs for ζ , ρ and $\gamma = r/\sigma$, and introduce an additional constraint: $r^{eq} \geq 0$. To satisfy this constraint, we let

$$\theta^{ids} = \begin{cases} 1.1 \theta^{eq}, & \text{for } \theta^{eq} < 0, \\ 0.9 \theta^{eq}, & \text{for } \theta^{eq} > 0, \end{cases} \quad (9)$$

so that for all cases, $\theta^{eq} > \theta^{ids}$ and hence, $r^{eq} > r \geq 0$. Other inputs are given by (7). The results computed with $\zeta > 1$ are consistent with those generated by Henderson (2007) and the complete market models. That is, the real option can be exercised and its value generally decreases with the individual investor's risk aversion. This conclusion also holds for $0 \leq \zeta \leq 1$, which is consistent with Henderson (2007)'s work. Interestingly, in Table 1 with $\zeta < 0$, although when ζ falls, the real option value increases, yet it is still far less than infinity, indicating that there are still possibilities for the investor to exercise the real option. We argue that Henderson (2007)'s result (of the real option value being infinitely high in this case), results from her assumption that the option has an infinite maturity and a fixed strike. Under these assumptions, a real option may be much more valuable as a protection against the downside risks than it would be of the strike being stochastic and positively correlated with the underlying project value.

Previous studies assume perfect hedging of all hedgeable risks involved in the project (as in Henderson, 2007). But given Adam and Fernando (2006)'s study that hedging practice is usually subjective and imperfect, we now investigate the impact on the add-on value generated by the investor's imperfect hedging activities. Recall our previous argument that the add-on option value arises from the disagreement between the individual investor and the market about the

	ζ	3					2.7				
	ρ	0.5	0.75	0.9	0.95	0.99	0.5	0.75	0.9	0.95	0.99
	θ^{eq}	-0.35	-0.46	-0.69	-0.96	-2.13	-0.29	-0.39	-0.59	-0.82	-1.81
	θ^{ids}	-0.38	-0.50	-0.76	-1.06	-2.34	-0.32	-0.42	-0.64	-0.90	-1.99
γ_{20}	θ_0^{eq}	40.66	27.11	22.59	21.40	20.54	40.56	27.04	22.53	21.34	20.49
	$\Phi_0 (10^{-8})$	9.35	7.19	4.77	3.43	1.56	7.94	6.11	4.01	2.89	1.32
γ_{10}	θ_0^{eq}	20.66	13.77	11.48	10.87	10.43	20.56	13.71	11.42	10.82	10.38
	$\Phi_0 (10^{-5})$	4.44	3.41	2.26	1.62	0.73	3.80	2.90	1.90	1.39	0.62
γ_5	θ_0^{eq}	10.66	7.11	5.92	5.61	5.38	10.56	7.04	5.87	5.56	5.33
	$\Phi_0 (10^{-3})$	1.76	1.35	0.88	0.64	0.29	1.50	1.14	0.76	0.54	0.25
γ_1	θ_0^{eq}	2.66	1.77	1.48	1.40	1.34	2.56	1.71	1.42	1.35	1.29
	Φ_0	0.13	0.10	0.06	0.05	0.02	0.11	0.08	0.05	0.04	0.02
γ_0	θ_0^{eq}	0.66	0.44	0.37	0.35	0.33	0.56	0.37	0.31	0.30	0.28
	Φ_0	0.46	0.36	0.23	0.17	0.08	0.39	0.30	0.20	0.14	0.06
	ζ	0.5					0.2				
	ρ	0.5	0.75	0.9	0.95	0.99	0.5	0.75	0.9	0.95	0.99
	θ^{eq}	0.09	0.11	0.17	0.24	0.53	0.14	0.18	0.28	0.38	0.85
	θ^{ids}	0.08	0.10	0.15	0.22	0.48	0.12	0.16	0.25	0.35	0.77
γ_{20}	θ_0^{eq}	39.87	26.58	22.15	20.98	20.13	39.78	26.52	22.10	20.94	20.09
	$\Phi_0 (10^{-8})$	2.37	1.81	1.18	0.85	0.39	3.76	2.90	1.91	1.37	0.62
γ_{10}	θ_0^{eq}	19.86	13.24	11.04	10.46	10.03	19.78	13.19	10.99	10.41	9.99
	$\Phi_0 (10^{-5})$	1.13	0.86	0.57	0.41	0.18	1.79	1.38	0.91	0.65	0.29
γ_5	θ_0^{eq}	9.87	6.58	5.48	5.19	4.98	9.78	6.52	5.44	5.15	4.94
	$\Phi_0 (10^{-4})$	4.43	3.36	2.24	1.61	0.72	7.11	5.41	3.59	2.54	1.16
γ_1	θ_0^{eq}	1.87	1.24	1.04	0.98	0.94	1.78	1.19	0.99	0.94	0.90
	Φ_0	0.03	0.02	0.02	0.01	0.01	0.05	0.04	0.03	0.02	0.01
γ_0	θ_0^{eq}	-0.14	-0.09	-0.08	-0.07	-0.07	-0.22	-0.14	-0.12	-0.11	-0.11
	Φ_0	0.12	0.09	0.06	0.04	0.02	0.19	0.14	0.09	0.07	0.03
	ζ	-0.2					-1.2				
	ρ	0.5	0.75	0.9	0.95	0.99	0.5	0.75	0.9	0.95	0.99
	θ^{eq}	0.21	0.27	0.41	0.58	1.28	0.38	0.50	0.76	1.06	2.34
	θ^{ids}	0.19	0.24	0.37	0.52	1.15	0.34	0.45	0.68	0.95	2.11
γ_{20}	θ_0^{eq}	39.68	26.45	22.04	20.88	20.04	39.41	26.27	21.89	20.74	19.90
	$\Phi_0 (10^{-8})$	5.63	4.33	2.85	2.03	0.93	10.39	7.84	5.23	3.77	1.70
γ_{10}	θ_0^{eq}	19.68	13.12	10.93	10.36	9.94	19.41	12.94	10.78	10.21	9.80
	$\Phi_0 (10^{-5})$	2.69	2.06	1.35	0.97	0.44	4.89	3.78	2.47	1.78	0.80
γ_5	θ_0^{eq}	9.68	6.45	5.38	5.09	4.89	9.41	6.27	5.23	4.95	4.75
	$\Phi_0 (10^{-3})$	1.06	0.81	0.54	0.39	0.17	1.94	1.49	0.98	0.70	0.32
γ_1	θ_0^{eq}	1.68	1.12	0.93	0.88	0.85	1.41	0.94	0.78	0.74	0.71
	Φ_0	0.08	0.06	0.04	0.03	0.01	0.14	0.11	0.07	0.05	0.02
γ_0	θ_0^{eq}	-0.32	-0.22	-0.18	-0.17	-0.16	-0.59	-0.40	-0.33	-0.31	-0.30
	Φ_0	0.28	0.21	0.14	0.10	0.05	0.50	0.39	0.26	0.18	0.08

Table 1: The add-on value of the option to invest for $\zeta = 3, 2.7, 0.5, 0.2, -0.2, -1.2$, $\rho = 0.5, 0.75, 0.9, 0.95, 0.99$ and $\gamma = 20, 10, 5, 1, 0$. θ^{ids} is set as a function of θ^{eq} (9) so that the constraint $r^{eq} > 0$ is satisfied. θ_0 is set equal to θ_0^{eq} .

project and attached real option value. Here we further argue that such argument applies to each risk factor involved in the project: the add-on value associated with a risk factor arises from the disagreement between the investor and the market regarding the premium for the factor. Hence, whether or not hedging the risk factor affects the add-on option value is generally equivalent with whether or not the individual investor has the same estimate for the risk premium as the market.

To see this, we suppose the individual investor may hedge the risk factor shared by the revenues and costs (1), \mathcal{B}_0 . We denote by h , $h \in [0, 1]$, the proportion of the exposure associated with \mathcal{B}_0 that is hedged; $h = 0$ represents that no hedging at all, whereas $h = 1$ represents a perfect hedging of \mathcal{B}_0 . Employing a hedging policy about \mathcal{B}_0 would change the expected growth rates of the operating cash flows, μ_i , $i = 1, 2$, and their volatilities, σ_i , to:

$$\mu_i^* = \mu_i - h \theta_0^{eq} \rho_i \sigma_i, \quad \sigma_i^* = \sigma_i \sqrt{1 - (h \rho_i)^2}, \quad (10)$$

respectively. Under the assumptions previously made to illustrate H_3 , *i.e.* $\rho = \rho_1 = \rho_2$, $\sigma = \sigma_1 = \sigma_2$, $\theta^{ids(eq)} = \theta_1^{(eq)} = \theta_2^{(eq)}$, the implied and required returns, r^{eq} and r (8), vary with the hedging policy as follows:

$$r^{eq} = \left[(1 - h) \theta_0^{eq} \rho + \theta^{eq} \sqrt{1 - \rho^2} \right] \sigma, \quad r = \left[(1 - h) \theta_0 \rho + \theta^{ids} \sqrt{1 - \rho^2} \right] \sigma. \quad (11)$$

Now, let us first consider the case where the individual investor agrees with the market about the risk premium for \mathcal{B}_0 but disagrees on the premium for the other two factors. More specifically, we set:

$$\theta_0^{eq} = \theta_0 = 2/3, \quad \theta^{eq} = 2/3, \quad \theta^{ids} = 0.$$

In this case, the add-on option value for a range of h is depicted by the left panel in Figure 9. Three surfaces computed with different inputs of the correlation ρ and volatility σ , are all flat with respect to h , indicating that the add-on option value is unaffected by h increasing from 0 and 1. By contrast, this relationship does not hold in the second case depicted in the right panel of Figure 9 when the individual investor disagrees with the market about the risk premium for \mathcal{B}_0 but agrees about the premia for the other two sources of risks with the parameters. Here we

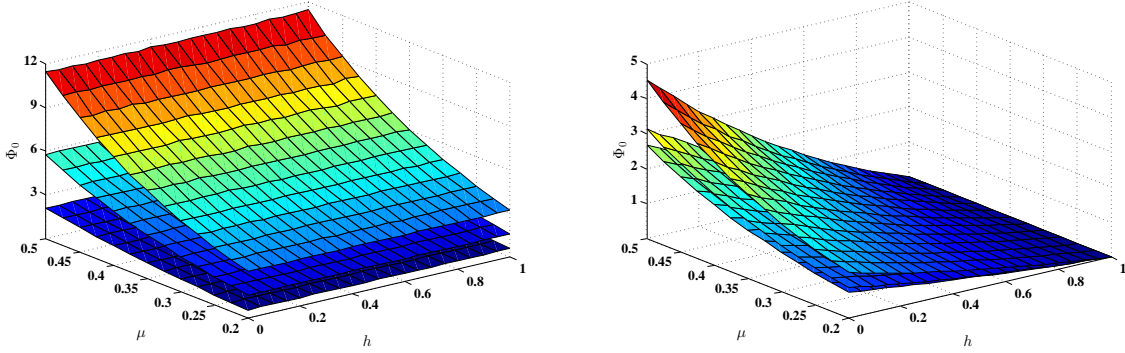


Figure 9: The real option value with different values of h and μ , $\mu := \mu_i, i = 1, 2$. The expected growth rates and volatilities of the operating cash flows are given by (10), and the implied and required returns are defined by (11). For the left panel, $\theta_0^{eq} = \theta_0 = 2/3$, $\theta_{eq} = 2/3$, $\theta^{ids} = 0$, whereas for the right panel, $\theta_0^{eq} = 2/3$, $\theta_0 = 0$, $\theta^{eq} = \theta^{ids} = 2/3$. Other inputs for the left panel include $\rho = 0.5, 0.5, 0.8$ and $\sigma = 0.5, 0.3, 0.3$ (from highest to lowest); similarly, for the right panel, $\rho = 0.5, 0.8, 0.5$ and $\sigma = 0.5, 0.3, 0.3$.

set

$$\theta_0^{eq} = 2/3, \quad \theta_0 = 0, \quad \theta^{eq} = \theta = 2/3.$$

All three surfaces decrease as h increases, indicating a monotonic and negative impact of hedging on the add-on value of the real option. These results demonstrate H_4 that the effect of hedging a source of risk involved in the project depends on the agreement between the individual investor and the market about the premium for this risk: if they agree, then hedging this risk should not affect the add-on option value attached to the project; otherwise, such hedging should reduce the option's add-on value.

4 Conclusions

In response to the ambiguous answers presented in the literature to the question if or not the real option value is incorporated in the underlying project value, we argue that the value of a real option has two components – that perceived by the market and that perceived by the individual investor. The first component is incorporated in the equilibrium project value and can be traded as part of the project if the project were tradable. It follows that the valuation of this component can employ the standard risk-neutral valuation techniques developed for financial option pricing. The second component is the add-on option value which corresponds to the

individual investor's flexibility to make his own optimal investment strategy in response to the market representative beliefs. It is a profit net of all cost of executing the strategy (*eg.* a positive NPV). We hence argue that the add-on value is subjective, and would converge to zero when the market is perfect.

This distinction also answers to other questions descendent to the ambiguity of the real option value, such as: would an investor pay to obtain a real option? Can the real option value be perfectly replicated? The first component of the real option value is unique to all investors who should pay for it. Only the financing efficiency and risk preferences of the market representative investor would affect this value. The second component, on the other hand, would also depend on the individual investor's risk preferences and efficiency, just like a project NPV; this also means that the add-on option value is non-replicable.

These arguments enable a model to be specified with a stochastic strike price (*i.e.* the market value of the project) under the physical measure. This is an intuitive development of classic real option valuation models, which tend to capture the add-on option value using a fixed strike price under the risk-neutral measure. Allowing for a stochastic option strike has the advantage that the add-on option value can be seen to approach zero as the market conditions approximate a perfect state. And, innovatively, we adopt a risk-adjusted measure under which all values are presented in PV terms. We also overcome the alleged difficulty of finding an appropriate discount rate for option pay-off by discounting the project pay-off first and then valuing the attached real option.

Within this framework, we study the parameter impacts on the add-on value of the option to invest, including expected return and risk aversion effect, volatility and idiosyncratic risk effects. Our simulation results about the implied and required return effect on the add-on value show that generally, the add-on value arises from a difference in financing efficiency and risk preferences between the individual investor and the market. When the individual investor is already extremely inefficient or risk averse compared with the market, the add-on value may start to fall if the investor keeps increasing his required return on the project. It then follows that the relationship between add-on option value and investor's risk preferences is not monotonic.

In general, the add-on value increases with cash flow volatilities. We hence argue that in addition to being a protection against downside risks as commonly argued in the literature, the real option also allows the investor to profit at lower costs when the volatility of operating costs increases. Increasing the idiosyncratic risk also increases the option's add-on value unless the investor is extremely inefficient or risk averse. This also holds when we set the idiosyncratic risk premia to be negative. Hence, an investor can profit strategically from bearing idiosyncratic risks even with the market discourages taking such risks.

Strong evidence has been documented by [Ang, Hodrick, Xing, and Zhang \(2006\)](#) that the market premia on idiosyncratic risks are significant. This, calls into question the wisdom of assuming that the investor in a real option valuation framework is well-diversified. It is equivalent to assuming that the investor is not exposed to idiosyncratic risks, or at least that he requires no idiosyncratic risk premium for bearing the risks. If the former assumption is used in a real option model, then its results would be rather questionable, in that the add-on value arising from idiosyncratic risks would be ignored entirely. If the investor does not require any idiosyncratic risk premium, then the idiosyncratic risk effect on the real option value needs to be properly understood, consistent with our previous argument.

We further show that hedging against a source of project risk would reduce the add-on option value if the individual investor disagrees with the market about the associated premium. Moreover, the hedging effect reveals the importance of investigating the idiosyncratic risk effect on the add-on value. In practice, the add-on value of a real option is more likely to be associated with idiosyncratic risks than with systematic risks. Systematic risks are so frequently traded that there is little chance that an individual investor would disagree with the market about the systematic risk premia. In other words, it is critical for the investor to design his investment policy in response to the idiosyncratic risks involved in the project, as it is more often the case that add-on value arises from bearing idiosyncratic risks than systematic risks. Also, it is not necessary to assume perfect hedging for the systematic risk in a real option valuation model because the hedging of such risk should not affect the investor's optimal investment policy.

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