

Discussion Paper

Measuring Macroeconomic Uncertainty: US Inflation and Output Growth

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Measuring Macroeconomic Uncertainty: US Inflation and Output Growth

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Abstract

We find that model estimates of the term structure of *ex ante* or perceived macro uncertainty are more in line with realized uncertainty than survey respondents' perceptions for both inflation and output growth. Survey estimates contain short-term variation in short-horizon uncertainty which is less evident in the model-based estimates. We consider the extent to which these short-term variations coincide with short-term movements in stock market uncertainty.

Keywords: Ex ante and ex post uncertainty, Macro and stock market uncertainty, MIDAS models.

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1 Introduction

There is now an accumulation of evidence suggesting that surveys provide more accurate forecasts than models, at least for nominal variables such as inflation, while for real variables the evidence is more mixed (see, for example, Ang, Bekaert and Wei (2007), Faust and Wright (2009, 2012) and Aiolfi, Capistrán and Timmermann (2011)). Survey forecasts might be expected to do well for a variety of reasons, including the timeliness of the information on which they draw. Faust and Wright (2009) consider the issue of timeliness in a comparison of the US Greenbook inflation and output growth forecasts to large dataset methods and simple univariate forecasting methods.

There is less evidence on the relative performance of survey expectations and models in terms of real-time forecasting of the *ex ante* uncertainty surrounding the future course of key macro aggregates, such as inflation and output growth. The effects of macroeconomic uncertainty on economic activity has long been of interest to economists, including whether surprises in uncertainty cause declines in output, or vice versa.¹ It is common to measure general uncertainty about the macroeconomic outlook using option-implied volatility estimates from stock market or exchange rate data, or survey-based data on the dispersion of forecasts or on consumer confidence.² Rather than attempting to measure general macroeconomic uncertainty our interest is in uncertainty more narrowly defined: uncertainty about the future course of inflation, and uncertainty about future output growth. This is because direct estimates of inflation and output growth uncertainty are provided by survey respondents' reported histograms, and one of our aims is to compare survey measures of uncertainty with model-based estimates.³ From a monetary policy perspective, inflation uncertainty is a key and distinct element of the general uncertainty about the future, which is likely to trigger a policy response.⁴

The relative performance of models and surveys at second-moment prediction has received little attention in the literature, notwithstanding the large literature comparing the two for first-moment prediction. The importance of data timeliness in the literature on first-

¹For example, Carroll (1996) considers the effects of uncertainty about labour income on households spending decisions, and Dixit and Pindyck (1994) and Bloom (2009) consider the effects on firms and their investment plans.

²Bloom (2009, Table 1, p.629) shows that stock market volatility is correlated with cross-sectional measures of uncertainty: the cross-sectional standard deviation of firms' pre-tax profit growth; a cross-sectional stock-return measure; the cross-sectional spread of industry productivity growth; and the dispersion of the Livingstone half-yearly survey forecasts of GDP.

³Of course the survey respondents may well base their forecasts on models, so the distinction is between mechanical model-based forecasts and forecasts which make use of model(s) and judgment to varying degrees.

⁴For example, the Bank of England's 'fan charts' reflect the uncertainty about the future course on inflation as well as providing information on the balance of risks to the central projection (see <http://www.bankofengland.co.uk/publications/Pages/inflationreport/default.aspx>). This informs the Bank's Monetary Policy Committee's interest rate decisions. Other central banks and governments are similarly influenced by the uncertainty concerning the inflation outlook.

moment prediction motivates the inclusion in our study of models which in principle allow the use of data that would have been available up to the point at which the corresponding survey return was made, so that the model and survey information sets are closely aligned in the time dimension.

The timings of the surveys and the horizons for which forecasts are provided determine the nature of the comparisons we report. To ensure a fair comparison, the models' outputs are manipulated to match the quantities which can be calculated from the survey responses. For example, the survey measures of forecast uncertainty relate to calendar-year annual inflation and year-on-year output growth made at horizons of (approximately) one-quarter up to eight quarters ahead. We show how estimates of these quantities can be obtained from popular forecasting models' outputs. In addition, the models are specified and estimated using the data which would have been available in real time, to match the surveys which are by their nature real time.

We compare the *ex ante* forecasts of uncertainty to *ex post* or realized uncertainty. Clements (2014) compares *ex ante* and *ex post* uncertainty for the survey forecasts, where the *ex ante* measure is the standard deviation of the reported histograms, and where the *ex post* measure is calculated using the survey point predictions once the actual realizations are known. He finds that *ex ante* uncertainty exceeds realized uncertainty for both inflation and output growth at within-year horizons. Motivated by these findings, our key question is whether models' *ex ante* forecasts of uncertainty more accurately reflect realized uncertainty.⁵ By and large we do find that the model estimates of *ex ante* uncertainty more closely track the decline in realized uncertainty as the forecast horizon shortens.

The plan of the rest of the paper is as follows. Section 2 defines the notions of *ex ante* and *ex post* uncertainty, and describes the survey data estimates. The nature of the survey data determines the nature of the forecast uncertainty estimates we require from the models, in order to allow a comparison of the survey and model forecasts. Section 3 describes the models. Section 3.1 outlines the calculation of *ex ante* uncertainty for the inflation forecasting model of Stock and Watson (2007), which has become a popular model for forecasting the level of inflation, and section 3.2 outlines the MIDAS models (MIXed DATA Sampling models, see e.g., Andreou, Ghysels and Kourtellos (2011)) which permit use of information up to the time of the survey return deadline, to assess the importance of the timeliness of the data. Section 4 reports the model estimates of the resolution of uncertainty as the forecast horizon shortens, and compares this with the surveys estimates. Section 5 compares the model and survey estimates of the time paths of uncertainty at various

⁵In the context of assessing DSGE model forecasts, Herbst and Schorfheide (2012) similarly assess whether the realized RMSEs are commensurate with what would be expected given the DSGE model's predictive distribution.

horizons. A number of extensions to the models are considered, including allowing for the onset of the Great Moderation (see, e.g., McConnell and Perez-Quiros (2000)) following the more turbulent 1970's. In section 6 we consider the relationship between forecasts of stock market volatility and perceived macro uncertainty, and the extent to which short-term variations are common to both. Section 7 concludes.

2 Measuring *ex ante* and *ex post* uncertainty

We define uncertainty in a natural way, as the variance of the unpredictable error in forecasting the future value of a variable. So given a target variable y_t , an information set \mathcal{I}_{t-h} , and a forecast $y_{t|t-h} = E(y_t | \mathcal{I}_{t-h})$, the unpredictable error is $e_{t|t-h} = y_t - y_{t|t-h}$, and uncertainty is:

$$E\left(e_{t|t-h}^2 | \mathcal{I}_{t-h}\right) \quad (1)$$

although we will often use the square root of this quantity. In practice the forecasts will be based on a given model, and different models will provide different estimates of the forecastable component, and therefore of the unpredictable error. The error from a particular model might be predictable using a model that draws on a wider information set, for example. Below we discuss modelling choices, and the information set in terms of real-time vintages of data.

A key distinction to be drawn is between *ex ante* uncertainty (henceforth, EAU) and *ex post* uncertainty (EPU) for a given model (or survey). The *ex post* estimates of (1) are based on sample second moments of the forecast errors, and so require the actual values. The *ex ante* estimates are made at time $t - h$. Survey estimates of EAU require probability distributions of the future values of the variables (usually reported as histograms) although disagreement has often been used as a proxy for uncertainty, beginning with Zarnowitz and Lambros (1987). Models provide estimates of EAU based on the in-sample fit of the model estimated on data through $t - h$, possibly supplemented with a time-varying conditional variance for the unexplained component of the model.

Because forecasting models will approximate the data generating process with varying degrees of accuracy, and because the process is likely to exhibit non-constancy over time, both the model and survey estimates of EAU and EPU may only poorly reflect uncertainty about the target variable. Survey participants draw on models but ‘use a variety of procedures to predict the major expenditure components of GNP, combine these predictions in nominal and real terms, and check and adjust the resulting forecasts for consistency with logic, theory, and the currently available information’ (as summarized by Zarnowitz and Braun (1993, p. 23)). That judgement is assigned an important role is also evident from

recent surveys of the ECB and US SPF forecasters (see ECB (2009) and Stark (2013), respectively). Of interest is whether survey respondents are more or less successful at assessing the uncertainty about the future. One might surmise that survey respondents' would bring useful extraneous information to the forecasting of uncertainty - relative to the mechanical operation of a model - especially during times of structural change in the economy. However, the literature on the psychology of judgement under uncertainty suggests there may be cognitive biases which might adversely affect the accuracy of the survey respondents' assessments.⁶

A possible disadvantage faced by models is that they are typically slow to react when circumstances change (see, e.g., Giordani and Söderlind (2003) in the context of forecasting inflation uncertainty). It seems likely that models estimated primarily on pre-Great Moderation data (i.e., on data before 1984) will tend to over-state EAU during the subsequent period of a lower underlying level of volatility of the economy. To a first approximation, the average level of unexplained variability over the estimation period will determine the model-estimate of EAU.⁷ We consider two strategies to account for the decline in volatility: using rolling windows for model estimation; and truncating the model estimation sample using the Sensier and van Dijk (2004) pre-test for a break in volatility. In addition to long-run changes in volatility, the regression models do not capture short-run changes in the predictable component of uncertainty. This shortcoming may be more important when estimating inflation EAU since there is strong evidence of time-varying volatility (see, e.g., Stock and Watson (2007)). We use a model that allows for smooth changes in the conditional variance over time, and we also draw on time variation in stock market uncertainty.

In order to motivate the form of the model uncertainty estimates we require to match the survey estimates, we next briefly explain the nature of the survey estimates.

2.1 SPF data

We compare our model estimates to uncertainty estimates using survey data, following Clements (2014). Clements (2014) uses the US Survey of Professional Forecasters (SPF) as it spans a long historical period compared to other surveys which provide similar information.⁸ It is a quarterly survey of macroeconomic forecasters of the US economy from 1968 to the present day, initially administered by the American Statistical Association (ASA) and the

⁶See, for example, O'Hagan, Buck, Daneshkhah, Eiser, Garthwaite, Jenkinson, Oakley and Rakow (2006, ch. 3) for a concise review of Kahneman and Tversky's 'heuristics and biases' research programme, and for a review of Hammond's cognitive continuum theory (O'Hagan *et al.* (2006, p.56)).

⁷This is true of the MIDAS regressions models. The UC-SV model considered below allows for changes in the variance over time.

⁸The Bank of England Survey of External Forecasters provides similar information for the UK, but only from beginning in 1996 (see, for example, Boero, Smith and Wallis (2012)), and since 1999 the ECB Survey of Professional Forecasters (SPF) covers the euro area (see, e.g., Garcia (2003)).

National Bureau of Economic Research (NBER) as the ASA-NBER Survey, and since June 1990 by the Philadelphia Fed as the SPF: see Zarnowitz (1969), Zarnowitz and Braun (1993) and Croushore (1993).

The SPF provides respondents' forecast distributions of the annual rate of output growth and the inflation rate, in the form of histograms. The histograms refer to the annual change from the previous year to the year of the survey, as well as of the survey year to the following year. As an example, consider the histogram forecasts of the annual output growth in 2005. We have eight histograms of this target, beginning with a forecast made in the first quarter of 2004 (a forecast of the following year relative to the survey quarter year), and ending with a forecast made in the fourth quarter of 2005 (a forecast of the survey quarter year on the previous year). We term these 8 to 1 step ahead forecasts. The histograms are reported by the middle of the middle month of the quarter, so this defines the cut off point for information for the models. We consider the aggregate survey histograms (averaging all available individual histograms at each point), and calculate uncertainty forecasts by fitting normal distributions to these histograms, following Giordani and Söderlind (2003).

Hence we need to construct forecasts of the uncertainty from the models that relate to the annual growth rate made at horizons of 1 to 8 quarters in advance. It is the form of the survey uncertainty estimates - namely, that they relate to the annual growth in a calendar year - that dictates the form of the model-based estimates we need to calculate.

3 Models for inflation and output growth uncertainty

3.1 Stock and Watson (2007) UCSV model

The Stock and Watson (2007) Unobserved Component Stochastic Volatility (UCSV) model has been shown to provide good inflation forecasts. The review of the forecasting performance of a wide range of inflation forecasting models by Stock and Watson (2008) shows that by and large the UCSV model is the best statistical forecasting model. At times when there are large departures from the NAIRU, models with economic activity variables might outperform the UCSV, but in 'normal times' economic variables generally contribute little to forecast accuracy. These comparisons are of point forecasts, whereas our interest is in forecasting the uncertainty about the annual rate of inflation, but suggest the UCSV might be a good candidate model.

The UCSV model decomposes quarterly inflation, π_t , into a permanent (τ_t) and a

transitory component (ε_t):

$$\begin{aligned}\pi_t &= \tau_t + \varepsilon_t \\ \tau_t &= \tau_{t-1} + \eta_t\end{aligned}\tag{2}$$

where $E(\eta_t) = 0$, $var(\eta_t) = \sigma_{\eta,t}^2$, $E(\varepsilon_t) = 0$, $var(\varepsilon_t) = \sigma_{\varepsilon,t}^2$, and $cov(\eta_t, \varepsilon_t) = 0$. We let $\varepsilon_t = \sigma_{\varepsilon,t}\zeta_{\varepsilon,t}$ and $\eta_t = \sigma_{\eta,t}\zeta_{\eta,t}$, where $\zeta_t = [\zeta_{\varepsilon,t}, \zeta_{\eta,t}]' \sim iiN(0, I_2)$, and model the variation in the disturbances by:

$$\begin{aligned}\ln \sigma_{\varepsilon,t}^2 &= \ln \sigma_{\varepsilon,t-1}^2 + v_{\varepsilon,t} \\ \ln \sigma_{\eta,t}^2 &= \ln \sigma_{\eta,t-1}^2 + v_{\eta,t}\end{aligned}\tag{3}$$

where $v_t = [v_{\varepsilon,t}, v_{\eta,t}]' \sim iiN(0, \gamma I_2)$.

The model allows the variances of the permanent and transitory components ($\sigma_{\eta,t}^2$ and $\sigma_{\varepsilon,t}^2$) to change over time, which enables the model to capture key aspects of postwar US inflation.⁹ The variance of the permanent component was large during the period 1970–83, but declined sharply in the mid-1980s, and has continued to decline thereafter. Thus the model captures the overall decline in inflation volatility over the forecast period via the decline in $\sigma_{\eta,t}^2$.

The UCSV models the quarterly rate of inflation, and provides forecasts of the conditional variance of the *quarterly* rate of inflation at various steps ahead. We need to construct forecasts of the year-on-year annual rate of inflation. We do this by approximating the annual growth rate z_t using the quarterly growth rates. Let y_t be (four hundred times) the first-difference of the quarterly log-level of the variable, Y_t (output, or the price deflator), i.e., $y_t = 400 \times \Delta Y_t$, then the *annual* growth rate, z_t , can be approximated by $z_t = \sum_{j=0}^6 w_j y_{t-j}$ where $w_j = \frac{j+1}{16}$ for $0 \leq j \leq 3$, and $w_j = \frac{7-j}{16}$ for $4 \leq j \leq 6$.

Note that z_t is a quarterly variable. The (approximations to) *calendar* year growth rates (which are comparable to the targets of the survey forecasts) are given by those z_t for which t corresponds to a fourth quarter of the year observation. This means that for a $h = 1$ quarterly horizon forecast of z_t , 6 of the 7 constituent quarterly growth rates required for the computation of the annual growth rate are known (i.e., y_{t-1} to y_{t-6}) with only the fourth quarter value of y_t needing to be forecast. Whereas for a $h = 7$ horizon forecast, for example, all the quarters (y_t to y_{t-6}) need to be forecast.

⁹See Cogley, Sargent and Surico (2012), who estimate the volatilities of the transient and permanent components of inflation for the US for the period 1791–2011 using a closely related model.

3.1.1 UCSV EAU

To illustrate the calculation of EAU for the UCSV, suppose $h = 1$. What does the model imply about the uncertainty surrounding π_t based on data through $t - 1$? From (2) we have $\pi_t = \tau_t + \varepsilon_t = \tau_{t-1} + \eta_t + \varepsilon_t$, so that the 1-step ahead forecast error is $\eta_t + \varepsilon_t$ (assuming the information set includes $I_{t-1} = \pi_{t-1}, \tau_{t-1}, \dots$), that is

$$\pi_t - E(\pi_t | I_{t-1}) = \eta_t + \varepsilon_t.$$

The conditional variance is:

$$\begin{aligned} \text{Var}(\pi_t | I_{t-1}) &= E(\eta_t^2 + \varepsilon_t^2 | I_{t-1}) \\ &= E(\sigma_{\eta,t}^2 \zeta_{\eta,t}^2 + \sigma_{\varepsilon,t}^2 \zeta_{\varepsilon,t}^2 | I_{t-1}) \\ &= E(\sigma_{\eta,t}^2 + \sigma_{\varepsilon,t}^2 | I_{t-1}). \end{aligned}$$

Using $E(\sigma_{\varepsilon,t}^2 | I_{t-1}) = \sigma_{\varepsilon,t-1}^2 \exp\left(\frac{\gamma}{2}\right)$ and $E(\sigma_{\eta,t}^2 | I_{t-1}) = \sigma_{\eta,t-1}^2 \exp\left(\frac{\gamma}{2}\right)$, the *ex ante* forecast-error variance of annual inflation when $h = 1$ is:

$$\sigma_{z,t|t-1}^2 = w_0^2 \left(\exp\left(\frac{\gamma}{2}\right) \left[\sigma_{\varepsilon,t-1}^2 + \sigma_{\eta,t-1}^2 \right] \right). \quad (4)$$

(The appendix A provides the details, and the derivation for a general h). It is *ex ante* in the sense that (4) only requires estimates of quantities ($\sigma_{\varepsilon,t-1}^2$ and $\sigma_{\eta,t-1}^2$) available at $t - 1$.

The h -step ahead *ex ante* forecast uncertainty for the UCSV model is given by:

$$\sigma_{z,n|n-h}^2 = \sigma_{\varepsilon,n-h}^2 \sum_{j=0}^{h-1} \left(w_j^2 \exp\left(\frac{(h-j)\gamma}{2}\right) \right) + \sigma_{\eta,n-h}^2 \sum_{j=0}^{h-1} \left(\left(\sum_{s=0}^j w_s \right)^2 \exp\left(\frac{(h-j)\gamma}{2}\right) \right). \quad (5)$$

Here the use of the n superscript in place of t emphasizes we use only those t corresponding to fourth quarters of the year, for which the weighted sum of quarterly inflation rates approximates a calendar-year annual inflation rate.

We let T be the number of years in the in-sample period, and P the number of years in the out-sample period, which correspond to the annual targets. Equation (5) provides the *ex ante* estimates of uncertainty for the target z_n , for $n = 4(T + 1), \dots, 4(T + P)$. To match the survey forecasts, $n = 4(T + 1)$ corresponds to the annual rate of inflation in 1983, and $n = 4(T + P)$ to 2010. We use real-time data throughout. Given that both the deflator and output are revised over time, this means using the then available data vintages. The timing of the surveys is such that advance estimates of the previous quarter values of output and inflation are known. To be precise, consider an $h = 1$ survey forecast. This is made in

the middle of the fourth quarter of the year, when the advance estimates of the national accounts for the third quarter have been issued. In our notation, this implies the UCSV is estimated on data through $n - h$ (the third quarter of the year) from the $n + 1 - h$ quarterly vintage (the Q4 vintage). So for the first forecast target (the annual rate of inflation in 1983), for $h = 1$, we estimate the UCSV model on quarterly data up to and including 1983:Q3 from the data vintage available at 1983:Q4. This provides estimates of $\sigma_{\varepsilon, n-1}^{2, n}$ and $\sigma_{\eta, n-1}^{2, n}$, where the superscripts denote the quarterly vintage date, and $n = 4(T + 1)$. These estimates are plugged into equation (5), which for $h = 1$ specializes to:

$$\sigma_{z, 4T+4|4T+3}^{2, 4T+4} = w_0^2 \left(\exp\left(\frac{\gamma}{2}\right) \left[\sigma_{\varepsilon, 4T+3}^{2, 4T+4} + \sigma_{\eta, 4T+3}^{2, 4T+4} \right] \right)$$

as the only unknown quarterly growth rate is for the fourth quarter, and this receives a weight of w_0 in z_t .

For $h = 2$, we estimate the model on date through 1983:Q2 from the data vintage available at 1983:Q3, and inflation uncertainty is given by:

$$\sigma_{z, 4T+4|4T+2}^{2, 4T+3} = \sigma_{\varepsilon, 4T+2}^{2, 4T+3} \left(w_0^2 \exp(\gamma) + w_1^2 \exp\left(\frac{\gamma}{2}\right) \right) + \sigma_{\eta, 4T+2}^{2, 4T+3} \left(w_0^2 \exp(\gamma) + (w_0 + w_1)^2 \exp\left(\frac{\gamma}{2}\right) \right).$$

The expression reflects the fact that the third and fourth quarter quarterly growth rates need to be forecast (i.e., π_{t-1} and π_t) and that these are weighted by w_1 and w_0 , respectively, and that the errors in forecasting these growth rates are necessarily correlated (the Appendix A provides the details).

In general, we can write (5) taking into account the use of real-time data for forecasting targets $n = 4(T + 1), \dots, 4(T + P)$ as:

$$\sigma_{z, n|n-h}^{2, n+1-h} = \sigma_{\varepsilon, n-h}^{2, n+1-h} \sum_{j=0}^{h-1} \left(w_j^2 \exp\left(\frac{(h-j)\gamma}{2}\right) \right) + \sigma_{\eta, n-h}^{2, n+1-h} \sum_{j=0}^{h-1} \left(\left(\sum_{s=0}^j w_s \right)^2 \exp\left(\frac{(h-j)\gamma}{2}\right) \right), \quad (6)$$

where the superscripts denote the quarterly vintage we employ to estimate the UCSV model and to compute both conditional variances.

So far we have disregarded the uncertainty emanating from the estimation of the conditional variances. We estimate the UCSV model by MCMC, following Stock and Watson (2007), and calibrate γ at 0.4 (a value supported by the estimates in Cogley *et al.* (2012)). The variances $\sigma_{\varepsilon, t}^2$ and $\sigma_{\eta, t}^2$ are the means of the posterior distribution. We can use the posterior distribution to compute upper and lower bounds for $\sigma_{z, n|n-h}^2$.

3.1.2 UCSV EPU

The computation of EPU employs the point forecasts. For the UCSV model, the point forecasts are simply the estimate of the trend. That is, for a horizon h , using data through $n - h$ from vintage $n + 1 - h$, the forecasts of $n - h + 1, \dots, n$ are τ_{n-h}^{n+1-h} . These forecasts of future quarters are combined with data on past inflation rates (appropriately weighted) to give the forecast for annual inflation as:

$$z_{n|n-h} = \tau_{n-h}^{n+1-h} \sum_{j=0}^{h-1} w_j + \sum_{j=h}^6 w_j \pi_{n-j}^{n+1-h} \quad \text{for } h < 7$$

$$z_{n|n-h} = \tau_{n-h}^{n+1-h} \sum_{j=0}^6 w_j = \tau_{n-h}^{n+1-h} \quad \text{for } h \geq 7$$

Given the P values of actual annual inflation and the model forecasts for each h we calculate the EPU as the RMSFE. We need an assumption on the vintage of data to use for the target z_n to compute forecast errors. We use the ‘second-release’ data vintage series as ‘actual values’, i.e., the vintage available two quarters after the reference quarter.

3.2 MIDAS models

We consider Mixed Data Sampling (MIDAS) regressions to exploit the information that is available in addition to the quarterly values of the series being forecast.¹⁰ MIDAS models might be expected to provide reasonable estimates of the term structure of EAU (i.e., when we take time averages), which we consider in section 4. In contrast with the UCSV model, as discussed in section 6, direct multi-step ahead forecasting with the MIDAS model does not readily permit time-variation in the model’s forecast-error variance, so the MIDAS model may be less successful than the UCSV at capturing variation over time in EAU. The MIDAS model assumes a constant variance of the disturbance over the in-sample period, and this is the basis of the forecast of uncertainty out-of-sample.

Any limitations that this might impose can be partially mitigated by the use of rolling estimation windows (as opposed to the use of a recursive scheme, for example), or truncating the in-sample period conditional on finding a break in variance. We also experiment with scaling the estimates by forecasts of stock market volatility when we consider time paths (section 5).

MIDAS models allow us to exploit the information content of monthly and daily data when computing model-based estimates of EAU and EPU. MIDAS models have been used by a number of authors to exploit daily and monthly data, including Ghysels and Wright (2009),

¹⁰Andreou *et al.* (2011) provides a recent review of MIDAS models. See also Ghysels, Santa-Clara and Valkanov (2006), Ghysels, Sinko and Valkanov (2007) and Clements and Galvão (2008).

Andreou, Ghysels and Kourtellis (2010) and Clements and Galvão (2008, 2013). The choice of monthly explanatory variables is guided by economic calendars such as Bloomberg¹¹, which describes a set of data releases identified as ‘market moving’. Most of these data releases refer to monthly measures of economic activity, such as industrial production, non-farm payroll (employment), PMI (purchasing managers index), retail sales, and housing activity. We elect to use the monthly predictors labeled as ‘market moving’ which are related to economic activity and which are available as real-time data vintages from 1982. The variables are listed in Table 1. Of these, only PMI is not subject to revisions.

We also include daily equity index (SP500) returns. This variable has been shown to incorporate the effect of macroeconomic news during the first month of the quarter (Gilbert, Scotti, Strasser and Vega (2010)), and to have predictive ability for future output growth (Andreou *et al.* (2010)) and output growth data revisions (Clements and Galvão (2013)).

3.2.1 Specification of MIDAS

As described previously, the fixed event is annual, but forecasts are computed quarterly, so we estimate MIDAS models using the following *quarterly* LHS variables:

$$\begin{aligned}
 q_t^h &= \sum_{i=0}^{h-1} \frac{1+i}{16} y_{t-i} \text{ for } h = 1, \dots, 4 \\
 q_t^h &= \sum_{i=0}^3 \frac{1+i}{16} y_{t-i} + \sum_{i=0}^{h-5} \frac{3-i}{16} y_{t-(4+i)} \text{ for } h = 5, \dots, 7 \\
 q_t^h &= z_t \text{ for } h \geq 8.
 \end{aligned} \tag{7}$$

Recall that y_t is the quarterly growth rate, $y_t = 400 \ln(Y_t/Y_{t-1})$ where Y is the quarterly level (the price deflator, or level of output). The definition of q_t^h comes from the quarterly growth rates approximation to the annual growth rate (described in section 3.1). For a given horizon h , (7) gives the quarters that need to be forecast weighted by their importance in approximating z_t . For $h = 1$, for example, from (7) $q_t^h = \frac{1}{16} y_t$. Suppose t is a fourth quarter of the year - then only the value for the fourth quarter of the year needs to be forecast. Note that q_t^h is a quarterly frequency variable. Our estimates of EAU and EPU only use the subset of forecasts for which $t \in n$, that is, for which t is a fourth quarter, but we are able to use all the quarters to estimate the unknown parameters of the models for q_t^h , discussed below.

Firstly, some notation. Let x_t^M denote a variable available at the monthly frequency, where x_t^M is the last month in quarter t , $x_{t-1/m}^M$ the penultimate month in quarter t , etc., and $m^M = 3$ (the number of months in a quarter). If x_t^M is released before y_t , as is generally

¹¹<http://www.bloomberg.com/markets/economic-calendar/>

the case, we can use ‘leads’ - typically for monthly data we are able to use $l^M = 1/3$ (one month). Consider firstly a model with a single monthly indicator variable and lags of the growth rate of the quarterly dependent variable:

$$\begin{aligned}
q_t^h &= \beta_0 + \beta_Q \sum_{i=0}^{p-1} w(\theta_Q, i) y_{t-h-i} \\
&\quad + \beta_M \sum_{i=0}^{p^M-1} w(\theta_M, i) x_{t-(3h/m^M)-(i/m^M)+l^M}^M + \varepsilon_t^h,
\end{aligned} \tag{8}$$

where we may use $p^M = pm^M$, where p is the maximum number of lags in quarters of the quarterly dependent variable. In the empirical application, we use $p = 8$ and estimate weighting functions for both the lags on the quarterly dependent variable and for the monthly indicator. As an example, suppose $l^M = 1/3$ and $h = 1$. Then (8) implies the use of data on the first month of the quarter t ($x_{t-2/3}$), as this would have been available to the survey respondents who file their returns by the middle of the middle month of quarter t .

For the lag weighting functions we use the beta function:¹²

$$\begin{aligned}
w(\theta; i) &= \frac{f(\theta; i)}{\sum_{j=1}^K f(\theta; j)} \\
f(\theta; i) &= \frac{\binom{k}{i}^{\theta_1-1} (1-k)^{\theta_2-1} \Gamma(\theta_1 + \theta_2)}{\Gamma(\theta_1) \Gamma(\theta_2)}; \quad k = i/(K+1).
\end{aligned}$$

In the case of quarterly lags, $K = p$, and in the case of monthly lags $K = pm^M$.

The model with both monthly and daily indicators is:

$$\begin{aligned}
q_t^h &= \beta_0 + \beta_Q \sum_{i=0}^{p-1} w(\theta_Q, i) y_{t-h-i} \\
&\quad + \beta_M \sum_{i=0}^{p^M-1} w(\theta_M, i) x_{t-(3h/m^M)-(i/m^M)+l^M}^M \\
&\quad + \beta_D \sum_{i=0}^{m^D-1} w(\theta_D, i) x_{t-(60h/m^D)-(i/m^D)+l^D}^D + \varepsilon_t^h,
\end{aligned} \tag{9}$$

where $m^D = 60$ (approximately number of business days in a quarter), so we use one quarter of daily data. Because there is no delay on the release of financial data, and forecasts are computed in the middle of the quarter, we use $l^D = 20/60 = 1/3$ (where 20

¹²We use the beta function as it is easier to choose initial values than for the exponential function, which is the other function that is often used. There is some evidence that the beta function is better for large K . Setting good initial values helps the NLS numerical optimization routines. Setting $\theta_1 = \theta_2 = 1$ gives uniform weights over all lags. When $\theta_2 > \theta_1 = 1$, the function is decreasing, implying that more recent information is given more weight. With this in mind, our initial values were set by fixing $\theta_1 = 1$, and varying θ_2 .

is the approximate number of business days in a month). This implies the use of daily information on the first month. (The convention is that x_t^D is the last day of quarter t . Therefore when $i = 0$ and $l^D = 1/3$, $x_{t-(60h/m^D)-(i/m^D)+l^D}^D = x_{t-1+1/3}^D$, indicating daily data for the first month of quarter t).

In addition to models with single monthly indicators as in the above, we also use a Factor MIDAS specification. We substitute x_t^M by f_t^M in equation (9). The factor f_t^M is obtained by principal components using the five monthly series in Table 1. Before the estimation of the factor and also of the MIDAS regressions, we transformed observed monthly levels to monthly quarterly growth rates at annual rates: $x_t^M = 400(\log(X_t^M) - \log(X_{t-m}^M))$, where X_t^M is the variable in levels. In the case of daily data, we apply a similar transformation to the original daily values in levels, namely $x_t^D = 400(\log(X_t^D) - \log(X_{t-m}^D))$.

3.2.2 MIDAS EAU

EAU is computed taking into account parameter uncertainty as explained in the appendix B. There we detail how we obtain $var(e_{n|n-h})$. From these, the EAU measures are given by:

$$\begin{aligned} var^{ea}(z_{n|n-h}) &= \frac{16}{\left(\sum_{i=1}^h i\right)} var(e_{n|n-h}) \text{ for } h = 1, \dots, 4 & (10) \\ var^{ea}(z_{n|n-h}) &= \frac{16}{\left(\sum_{i=1}^4 i + \sum_{i=0}^{h-5} (3-i)\right)} var(e_{n|n-h}) \text{ for } h = 5, 6 \\ var^{ea}(z_{n|n-h}) &= var(e_{n|n-h}) \text{ for } h = 7, 8. \end{aligned}$$

The scaling converts the quarterly $var(e_{n|n-h})$ estimates to annual rates, to match the survey quantities. The estimates of EAU are the sample averages of the $var^{ea}(z_{n|n-h})$ over n , where n indexes fourth quarter of the year.

3.2.3 MIDAS EPU

The calculation of EPU requires point forecasts of the annual growth rate z_n for $n = 4(T+1), \dots, 4(T+P)$. Using the forecasts, $q_{n|n-h}^h$, of the unknown quarterly components, q_n^h , we compute the forecasts of the calendar year growth rates, $z_{n|n-h}$, as:

$$z_{n|n-h} = q_{n|n-h}^h + \sum_{j=h}^6 w_j y_{n-j}, \quad (11)$$

for $h = 1, \dots, 6$ and as before $w_j = \frac{j+1}{16}$ for $0 \leq j \leq 3$, and $w_j = \frac{7-j}{16}$ for $4 \leq j \leq 6$. The EPU $var^{ep}(z_{n|n-h})$ is computed using mean of the squared forecast errors over the P observations

of the out-of-sample period for each h , i.e., $P^{-1} \sum_{n=1}^P (z_n - z_{n|n-h})^2$.

We estimate the MIDAS regressions using the vintages that would have been available to the survey respondents at each point in time. This means we can write the MIDAS regression using superscripts to denote vintage date in quarters as:

$$\begin{aligned}
q_t^{h,n+1-h} &= \beta_0 + \beta_Q \sum_{i=0}^{p-1} w(\theta_Q, i) y_{t-h-i}^{n+1-h} \\
&\quad + \beta_M \sum_{i=0}^{p^M-1} w(\theta_M, i) x_{t-(3h/m^M)-(i/m^M)+l^M}^{M,n+1-h+l^M} \\
&\quad + \beta_D \sum_{i=0}^{m^D-1} w(\theta_D, i) x_{t-(60h/m^D)-(i/m^D)+l^D}^D + \varepsilon_t^h,
\end{aligned}$$

for forecasting origins $n = 4(T + 1), \dots, 4(T + P)$ using quarterly observations $t = \dots, n - h - 1, n - h$. Note that data on the financial variables is not subject to revision (so these variables are not super-scripted), and that the lead operator l^M applies to the vintage of the monthly indicator (to show that we use more timely monthly vintages to estimate the model in addition to the quarterly vintages on y).

4 Forecasting the term structure of inflation and output growth uncertainty

The results in this section and the next (section 5) are based on the $P = 28$ annual targets from 1983 to 2010, and forecast horizons, $h = 1, \dots, 8$, to match the survey data. The real-time data on GDP and the GDP deflator is taken from the Real Time Data Set for Macroeconomists (RTDSM) maintained by the Federal Reserve Bank of Philadelphia (see Croushore and Stark (2001)). Details of the data sources for the monthly predictors in the MIDAS models are given in Table 1.

In this section we compare the term structure of uncertainty - how uncertainty varies with the horizon - for the survey and models. In so doing we average the estimates across the P annual targets. Figure 1 depicts the consensus survey EAU and EPU measures.¹³ The EAU for both variables are notably flatter in comparison with EPU, in the sense that EAU declines more slowly than EPU in h , and EAU exceeds EPU for both variables at within-year horizons ($h \leq 4$). These results can be interpreted as showing professional forecasters are under-confident (i.e., over-estimate uncertainty) at within year horizons for output growth and (at all horizons) for inflation.

Figure 2 presents the uncertainty measures for the UCSV model for inflation. We

¹³These are similar to the results reported in Clements (2014).

compute EAU and EPU from the model estimates and forecasts as described in section 3.1, using real-time data vintages, and hence mimicking the inflation data that would have been available to the survey participants at each forecast origin. For EAU the upper and lower one-standard-deviation bands for EAU are also plotted. In stark contrast to the survey results, EAU measured with UCSV under-estimates EPU except for $h = 1$, and at $h = 1$ is broadly equal to EPU, and less than a third of the survey EPU value. The intervals indicate marked uncertainty about the posterior mean estimates, so that only for $h = 3$ and 4 is EPU outside a \pm one standard deviation band about the EAU curve. The EAU curve is much steeper than the survey EAU curve, indicating the UCSV estimates capture the decline in uncertainty as the horizon shortens that is missed by the professional forecasters.

Figure 3 presents EAU and EPU measures from MIDAS models with different sets of explanatory variables. We consider models consisting of each one of the five monthly indicators described in table 1, using equation (8). Then we augment each of the five models with a daily indicator, as in equation (9), where the daily indicator is the SP500 daily return. Figure 3 summarizes the results, recording the maximum and the minimum EAU and EPU values for these models. In addition, we estimate a model with a factor computed from the 5 monthly variables, and include the daily financial variable. The dot denotes the uncertainty estimates for this model (which we term the MIDAS_FD below, to signify the factor (F) and daily data (D)). As in Figure 2, we use data from the vintage $n + 1 - h$ to estimate the model for each forecast origin and horizon. However, Figure 3 presents results for rolling windows of $4T - h - p$ observations, where $T = 25$ years. The use of rolling instead of recursive windows of data has almost no effect on the estimates of EPU, but in general reduces the estimates of EAU as the model is more closely tuned to the less volatile Great Moderation period.

Figure 3 broadly confirms the inflation results for UCSV reported in figure 2. The MIDAS models generate estimates of EAU that on average decrease as the horizon shortens in a way which is broadly compatible with EPU. The MIDAS_FD is among the more accurate models (a low EPU). Although there is always a better monthly indicator model than the MIDAS_FD, the best indicator depends on the horizon, and the factor model is never much worse. As a consequence, in the remainder of the paper we employ the MIDAS_FD specification as the representative MIDAS model to measure uncertainties.

In summary, the term structure of the survey measures of EAU is flatter than those of the UCSV and MIDAS model estimates of EAU, and of the estimates of EPU,¹⁴ for both variables.

¹⁴By and large, the term structure of EPU is similar whether estimated from model or survey forecasts. The key differences are between the EAU estimates.

5 The path of inflation and output growth uncertainty

In this section we look at how EAU changes over time for the survey and models. Figure 4 presents the time series of survey EAU for $h = 1, 4$ and 8 for each event date (top panels). The bottom left panel of Figure 4 presents output growth uncertainty estimates from the factor MIDAS model (9) estimated with rolling windows of data. For inflation, the bottom right panel of the figure displays estimates from the UCSV. In agreement with the survey estimates, the models capture the overall decline of EAU over time, and the upward movements towards the end of the sample. However they exhibit less short-term variability, and there are no cross-overs whereby uncertainty at shorter horizons exceeds that at longer horizons. In the case of output growth, for example, the survey EAU for $h = 1$ is larger than for $h = 8$ for calendar year 2009. Professional forecasters perceptions of uncertainty exhibit sensitivity to the economic environment at the time they forecast

5.1 Breaks and EAU

As discussed in Section 2, our use of rolling estimation enables the model EAU estimates to better track the reduction in underlying volatility documented by McConnell and Perez-Quiros (2000) and Sensier and van Dijk (2004), *inter alia*. However, for abrupt, one-off changes in volatility, approaches other than rolling windows may be preferable, and we use the testing procedure of Sensier and van Dijk (2004) to identify breaks in the conditional variance, and to estimate the date of the break. Their supWald statistic for a break in the variance is applied to the MIDAS disturbances, using p -values computed as in Hansen (1997).¹⁵ If we find a break at the 5% significance level, we only use the observations after the break to compute $var(e_{n|n-h})$, using the formulae in Appendix B, but with estimated parameters computed with the full sample.

Figure 5 presents three sets of MIDAS model estimates of EAU for $h = 1, 4, 8$: i) estimates computed with rolling windows of data; ii) estimates based on recursive estimation; and iii) estimates using only post-break observations when a break is found (labelled `rec_b` in the figure). The results indicate the existence of breaks in the conditional variance (when the estimates from i) and iii) deviate), and show the uncertainty estimates from rolling estimation declining over time (relative to the recursive estimates) as increasing weight is accorded to the post-break data. A break in the variance is detected for both series around 1992 for $h = 1$, and later on for the longer horizons. It may be harder to detect a break at longer horizons because the serial correlation induced by the overlapping nature of the multi-period forecasts may reduce the power of the test. The figure indicates that both es-

¹⁵We remove the first $0.15(n-h)$ observations and the last $0.15(n-h)$ from the grid to search for break dates.

estimates based on rolling windows, and those based on pre-tests for breaks, tend to indicate lower levels of uncertainty compared to using recursive estimation. Pre-testing suggests markedly lower output growth EAU in the 1990s at $h = 1$ in particular.

Figure 6 directly compares the survey estimates of EAU with model estimates that allow for changes in the conditional variance (the MIDAS_FD(rec_b) model for output growth, and this and the UCSV model for inflation). Some key differences remain. Firstly, the survey forecasts exhibit far more variability at $h = 1$, and the average level of volatility is markedly higher, especially for inflation. Secondly, for output growth, at a year ahead ($h = 4$) the time profiles of the survey and model estimates of EAU are broadly in line, whereas at $h = 8$ the average level of the uncertainty indicated by the survey is markedly lower than that of the model. Thirdly, year-ahead inflation uncertainty perceived by the survey respondents remains high throughout the last two decades compared to the model estimates. The two year ahead survey forecasts of inflation uncertainty are reasonably well tracked by the UCSV model

Table 2 summarizes the results obtained so far in terms of the correlations of the model estimates with the survey measure of EAU. (For now we ignore the last 2 rows of the table for each variable). The correlations for output growth are generally in the range of 50-80%, tending to be at the higher end for the medium horizons. For inflation, the model and survey forecasts are generally highly correlated. The UCSV model correlations are in excess of 80% at the longer horizons, but even at the shortest horizons are 66%. The MIDAS models perform similarly to the UCSV model and provide a reasonable match to the survey estimates for inflation.

6 EAU and Stock market volatility

In this section we consider the relationship between macro uncertainty (specifically, inflation uncertainty, and output growth uncertainty) and stock market uncertainty, given the perception that different measures of uncertainty tend to move together (see, e.g., Bloom (2009, 2013)). We examine whether stock market volatility might inform the survey respondents' perceptions of inflation and output growth uncertainty, and help explain the excess variability of the survey estimates (relative to the model estimates).

We also consider whether scaling the model estimates of inflation and output growth uncertainty by forecastable changes in stock market volatility results in a closer match to the survey estimates. We do this because a common way of modelling time-varying volatility is not applicable. The standard way of modelling short-term variation in volatility is to specify a (G)ARCH or a stochastic volatility model for the disturbance of the regression model. This is the approach taken by Jurado, Ludvigson and Ng (2013), for example, in a related

context. This is appropriate when it is reasonable to assume that the forecasting model has serially uncorrelated errors. However, by construction the way in which multi-step forecasts are generated for the MIDAS models induces serial correlation in the models' errors.¹⁶ Our solution is to allow EAU to vary with predicted stock market volatility. That we might be able to use predicted stock market volatility to model the conditional variation in the model EAU estimates is suggested by Bloom (2009, 2013), who uses stock market volatility as a measure of macroeconomic uncertainty. We scale the models' EAU estimates by estimates of 'excess' predicted stock market volatility at the relevant horizons. So if the forecast of stock market volatility is for a high value relative to its underlying level, we scale up the MIDAS model forecasts of inflation and output uncertainty. We compute predictions of stock market volatility by fitting MIDAS models to observed quarterly measures of market volatility.

The observed measure of stock market volatility (SMV) is constructed from daily SP500 data since 1959.¹⁷ Denote by V_t the standard deviation of daily returns over the last 240 business days starting from the last business day of quarter t . Daily returns are calculated as $100 \left[\ln(P_t) - \ln(P_{t-(1/m^D)}) \right]$. Then we estimate a MIDAS model for each h using daily squared returns as regressors, and using the same leads for the MIDAS models of the macro variables, but using lags up to one year:

$$V_t^h = \beta_0 + \beta_D \sum_{i=0}^{pm^D-1} w(\theta_D, i) sr_{t-(60h/m^D)-(i/m^D)+l^D}^D + \varpi_t^h,$$

where $sr_t^D = \left[100(\ln(P_t) - \ln(P_{t-(1/m^D)})) \right]^2$ and $pm^D = 240$ ($p = 4$ and $m^D = 60$). Based on the estimates of the above model, we compute $V_{n|n-h}^h$, the h -step ahead prediction of SMV.

To scale the model estimates, we let $var(e_{n|n-h})$ denote the average *ex ante* variance of the macro variable (inflation or output growth) over the sample from $n - h - w + 1$ to $n - h$ for a rolling estimation scheme with window size w . Details on how we compute $var(e_{n|n-h})$ are given in Appendix B. We measure how far predicted SMV is from a local historical average as:

$$v_{n|n-h}^h = \frac{V_{n|n-h}^h}{\left(\frac{1}{w} \sum_{t=n-h-w+1}^{n-h} V_t^h \right)},$$

and term this predicted excess SMV. We then scale the macro measures using $v_{n|n-h}^h$, so

¹⁶Carriero, Clark and Marcellino (2014) employ a MIDAS model with stochastic volatility, but only consider one-step horizons, so that serial correlation is not an issue.

¹⁷We cannot use the VIX as measure of stock market volatility because the time series is too short (only available from 1990) for our purposes.

that:

$$\text{var}(\epsilon_{n|n-h}^h) = \text{var}(e_{n|n-h}) v_{n|n-h}^h.$$

The estimates $\text{var}(\epsilon_{n|n-h}^h)$ allow for two sources of variation in macro EAU.¹⁸ The first is time-variation in the predictability of the macro variable at horizon h (captured by $\text{var}(e_{n|n-h})$, which is computed using the residuals of the MIDAS_FD model estimated with rolling windows of data). The second captures a component correlated with changes over time in predicted stock market excess volatility.

The bottom two rows of table 2 record the correlations of predicted SMV ($V_{n|n-h}^h$) with survey EAU for output growth and inflation, and the correlations of the scaled factor MIDAS model (MIDAS_FD: roll + vol) estimates with the survey estimates. Predicted SMV is not correlated with inflation EAU, but the correlations between SMV and survey output growth uncertainty at horizons of one and two quarters ahead are around one half. For these first two horizons, the correlations between the survey EAU and the rolling window MIDAS uncertainty estimates scaled by the predicted SMV are on a par with the correlations between survey EAU and the model estimates when we permit changes in conditional macro volatility by allowing for breaks in the variance (*rec_b*). This is consistent with professional forecasters being influenced by the outlook for financial markets when they report their assessments of the outlook for output growth.

Figure 7 supplements the information provided by the correlations. It includes a plot of predicted SMV (in basis points) for three horizons. The time axis denotes the target period, which is being forecasted either one quarter ($h = 1$), one year ($h = 4$) or two years ahead ($h = 8$). As would be expected, the time-variation in predicted SMV is much less at the two-year horizon¹⁹.

Figure 7 also presents the time series of the model EAU for output growth computed using rolling windows and scaled with conditional volatility as described above. The plot indicates years for which year-ahead volatility exceeds two year ahead volatility, showing that scaling by SMV results in the macro uncertainty measure becoming more sensitive to the current outlook. For example, the forecast of the year 2009 based on information available in 2009:M2 ($h = 4$) indicates far greater uncertainty than was apparent in the two-year ahead forecast based on 2008:M2. In the absence of scaling by predicted SMV the model uncertainty estimates are monotonically increasing in h across all forecast targets (see figure 4).

¹⁸Note that the model EAU is then obtained by annualising $\text{var}(\epsilon_{n|n-h}^h)$ as in equation (10).

¹⁹For a stationary process, current information will become less important as the forecast horizon increases, and we would expect the conditional forecast error variances to be close to the unconditional error variance of the process (see, e.g., Baillie and Bollerslev (1992) for a discussion in the context of AR processes with ARCH errors).

7 Conclusions

The conventional wisdom is that agents' probability assessments tend to be overconfident in the sense that they under-estimate the uncertainty they face (as documented for example in the literature on behavioral economics and finance: see the surveys by Rabin (1998) and Hirshleifer (2001)). The evidence provided by Clements (2014) for a survey of professional forecasters of the US macro-economy indicates that the respondents are under-confident at within-year horizons in their assessments of the uncertainty associated with the outlook for both inflation and output growth.

We find that more accurate assessments of uncertainty would have been possible in real-time had the survey participants used the forecasting models available nowadays. Generally, the models indicate a decline in *ex ante* forecast uncertainty as the horizon shortens, matching the decline in *ex post* uncertainty: the survey estimates in contrast have *ex ante* uncertainty staying at a relatively high level. The other key difference between the model and survey estimates is that the former indicate a good deal of variation over time - especially at short horizons - which is less apparent in the model estimates.

We show that the survey measure of output growth uncertainty is related to stock market uncertainty, but that inflation uncertainty and stock market uncertainty appear to have different determinants. A measure of predicted excess stock market volatility is used to model conditional variation in our MIDAS model measures of output growth uncertainty. This helps to bring the model measure more in line with the survey measure at short horizons. This is consistent with the view that survey respondents' perceptions of macro (output growth) are influenced by stock market developments, especially at short horizons up to half a year ahead, but we leave consideration of the causal relations between the different uncertainty variables for future research.

8 Appendix

8.1 Computation of EAU with the UCSV model

Suppose $h = 1$. Based on information through $t - 1$, we need to forecast π_t , the quarterly inflation rate in Q4 of year t .

From $\pi_t = \tau_t + \varepsilon_t = \tau_{t-1} + \eta_t + \varepsilon_t$, the forecast error is $\eta_t + \varepsilon_t$ (assuming the information set includes $I_{t-1} = \pi_{t-1}, \tau_{t-1}, \dots$), that is

$$\pi_t - E(\pi_t | I_{t-1}) = \eta_t + \varepsilon_t.$$

The conditional variance is:

$$\begin{aligned}
\text{Var}(\pi_t | I_{t-1}) &= E(\eta_t^2 + \varepsilon_t^2 | I_{t-1}) \\
&= E(\sigma_{\eta,t}^2 \zeta_{\eta,t}^2 + \sigma_{\varepsilon,t}^2 \zeta_{\varepsilon,t}^2 | I_{t-1}) \\
&= E(\sigma_{\eta,t}^2 + \sigma_{\varepsilon,t}^2 | I_{t-1}).
\end{aligned}$$

The first equality holds because $E(\eta_t \varepsilon_t | I_{t-1}) = 0$. The last line uses e.g., $E(\sigma_{\eta,t}^2 \zeta_{\eta,t}^2 | I_{t-1}) = E(\sigma_{\eta,t}^2 | I_{t-1}) E(\zeta_{\eta,t}^2 | I_{t-1}) = E(\sigma_{\eta,t}^2 | I_{t-1})$ since $\sigma_{\eta,t}$ and $\zeta_{\eta,t}$ are independent.

From (3), $\sigma_{\varepsilon,t}^2 = \sigma_{\varepsilon,t-1}^2 \exp(v_{\varepsilon,t})$, so $\sigma_{\varepsilon,t}^2$ is conditionally log-normal 1-step ahead, so that:

$$E(\sigma_{\varepsilon,t}^2 | I_{t-1}) = \sigma_{\varepsilon,t-1}^2 \exp\left(\frac{\gamma}{2}\right).$$

Similarly for $\sigma_{\eta,t}^2$:

$$E(\sigma_{\eta,t}^2 | I_{t-1}) = \sigma_{\eta,t-1}^2 \exp\left(\frac{\gamma}{2}\right).$$

Then the *ex ante* forecast-error variance of annual inflation when $h = 1$ is:

$$\sigma_{z,t|t-1}^2 = w_0^2 \left(\exp\left(\frac{\gamma}{2}\right) \left[\sigma_{\varepsilon,t-1}^2 + \sigma_{\eta,t-1}^2 \right] \right)$$

Estimating the model through $t - 1$ provides $\sigma_{\varepsilon,t-1|t-1}^2$ and $\sigma_{\eta,t-1|t-1}^2$, which replace $\sigma_{\varepsilon,t-1}^2$ and $\sigma_{\eta,t-1}^2$, so that the estimated *ex ante* forecast uncertainty is:

$$\sigma_{z,t|t-1}^2 = w_0^2 \left(\exp\left(\frac{\hat{\gamma}}{2}\right) \left[\sigma_{\varepsilon,t-1|t-1}^2 + \sigma_{\eta,t-1|t-1}^2 \right] \right)$$

Two-steps ahead

For $h = 2$ forecasts of z_t , we require:

$$\begin{aligned}
\pi_t - E(\pi_t | I_{t-2}) &= \eta_{t-1} + \eta_t + \varepsilon_t \\
\pi_{t-1} - E(\pi_{t-1} | I_{t-2}) &= \eta_{t-1} + \varepsilon_{t-1}
\end{aligned}$$

$$\begin{aligned}
\sigma_{z,t|t-2}^2 &= \text{Var}(w_0 [\eta_{t-1} + \eta_t + \varepsilon_t] + w_1 [\eta_{t-1} + \varepsilon_{t-1}] | I_{t-2}) \\
&= \text{Var}(w_0 \varepsilon_t + w_1 \varepsilon_{t-1} + w_0 \eta_t + (w_0 + w_1) \eta_{t-1} | I_{t-2}) \\
&= E(w_0^2 \sigma_{\varepsilon,t}^2 + w_1^2 \sigma_{\varepsilon,t-1}^2 + w_0^2 \sigma_{\eta,t}^2 + (w_0 + w_1)^2 \sigma_{\eta,t-1}^2 | I_{t-2}) \\
&= E(w_0^2 \sigma_{\varepsilon,t}^2 + w_0^2 \sigma_{\eta,t}^2 | I_{t-2}) + E(w_1^2 \sigma_{\varepsilon,t-1}^2 + (w_0 + w_1)^2 \sigma_{\eta,t-1}^2 | I_{t-2}) \quad (12)
\end{aligned}$$

The 2-step ahead expectations are evaluated as follows. For $E(\sigma_{\varepsilon,t}^2 | I_{t-2})$ we use:

$$\begin{aligned}
E(\sigma_{\varepsilon,t}^2 | I_{t-2}) &= E[E(\sigma_{\varepsilon,t}^2 | I_{t-1}) | I_{t-2}] \\
&= E\left[\sigma_{\varepsilon,t-1}^2 \exp\left(\frac{\gamma}{2}\right) | I_{t-2}\right] \\
&= \sigma_{\varepsilon,t-2}^2 \exp\left(\frac{\gamma}{2}\right) \exp\left(\frac{\gamma}{2}\right) \\
&= \sigma_{\varepsilon,t-2}^2 \exp(\gamma)
\end{aligned}$$

and similarly $E(\sigma_{\eta,t}^2 | I_{t-2}) = \sigma_{\eta,t-2}^2 \exp(\gamma)$. Substituting into (12) yields:

$$\begin{aligned}
\sigma_{z,t|t-2}^2 &= w_0^2 \exp(\gamma) (\sigma_{\varepsilon,t-2}^2 + \sigma_{\eta,t-2}^2) + w_1^2 \sigma_{\varepsilon,t-2}^2 \exp\left(\frac{\gamma}{2}\right) + (w_0 + w_1)^2 \sigma_{\eta,t-2}^2 \exp\left(\frac{\gamma}{2}\right) \\
&= \sigma_{\varepsilon,t-2}^2 \left(w_0^2 \exp(\gamma) + w_1^2 \exp\left(\frac{\gamma}{2}\right)\right) + \sigma_{\eta,t-2}^2 \left(w_0^2 \exp(\gamma) + (w_0 + w_1)^2 \exp\left(\frac{\gamma}{2}\right)\right)
\end{aligned}$$

Three-steps ahead

For $h = 3$ forecasts of z_t , we require:

$$\begin{aligned}
\pi_t - E(\pi_t | I_{t-3}) &= \eta_{t-2} + \eta_{t-1} + \eta_t + \varepsilon_t \\
\pi_{t-1} - E(\pi_{t-1} | I_{t-3}) &= \eta_{t-2} + \eta_{t-1} + \varepsilon_{t-1} \\
\pi_{t-2} - E(\pi_{t-2} | I_{t-3}) &= \eta_{t-2} + \varepsilon_{t-2}
\end{aligned}$$

$$\begin{aligned}
\sigma_{z,t|t-3}^2 &= \text{Var}(w_0 [\eta_{t-2} + \eta_{t-1} + \eta_t + \varepsilon_t] + w_1 [\eta_{t-2} + \eta_{t-1} + \varepsilon_{t-1}] + w_2 [\eta_{t-2} + \varepsilon_{t-2}] | I_{t-3}) \quad (13) \\
&= \text{Var}([w_0 + w_1 + w_2] \eta_{t-2} + [w_0 + w_1] \eta_{t-1} + w_0 \eta_t + w_0 \varepsilon_t + w_1 \varepsilon_{t-1} + w_2 \varepsilon_{t-2} | I_{t-3}) \\
&= E(w_0^2 \sigma_{\varepsilon,t}^2 + w_1^2 \sigma_{\varepsilon,t-1}^2 + w_2^2 \sigma_{\varepsilon,t-2}^2 + w_0^2 \sigma_{\eta,t}^2 + (w_0 + w_1)^2 \sigma_{\eta,t-1}^2 + (w_0 + w_1 + w_2)^2 \sigma_{\eta,t-2}^2 | I_{t-3})
\end{aligned}$$

Note that

$$\begin{aligned}
E(\sigma_{\varepsilon,t}^2 | I_{t-3}) &= E[E(\sigma_{\varepsilon,t}^2 | I_{t-2}) | I_{t-3}] \\
&= E[\sigma_{\varepsilon,t-2}^2 \exp(\gamma) | I_{t-3}] \\
&= \sigma_{\varepsilon,t-2}^2 \exp\left(\frac{3\gamma}{2}\right)
\end{aligned}$$

and more generally

$$E(\sigma_{\varepsilon,t}^2 | I_{t-h}) = \sigma_{\varepsilon,t-h}^2 \exp\left(\frac{h\gamma}{2}\right)$$

Then (14) becomes:

$$\begin{aligned}\sigma_{z,t|t-3}^2 &= \sigma_{\varepsilon,t-3}^2 \left(w_0^2 \exp\left(\frac{3\gamma}{2}\right) + w_1^2 \exp(\gamma) + w_2^2 \exp\left(\frac{\gamma}{2}\right) \right) \\ &\quad + \sigma_{\eta,t-3}^2 \left(w_0^2 \exp\left(\frac{3\gamma}{2}\right) + (w_0 + w_1)^2 \exp(\gamma) + (w_0 + w_1 + w_2)^2 \exp\left(\frac{\gamma}{2}\right) \right)\end{aligned}$$

h -steps ahead

For h -steps ahead the general formula is:

$$\sigma_{z,t|t-h}^2 = \sigma_{\varepsilon,t-h}^2 \sum_{j=0}^{h-1} \left(w_j^2 \exp\left(\frac{(h-j)\gamma}{2}\right) \right) + \sigma_{\eta,t-h}^2 \sum_{j=0}^{h-1} \left(\left(\sum_{s=0}^j w_s \right)^2 \exp\left(\frac{(h-j)\gamma}{2}\right) \right)$$

8.2 Computation of EAU with estimated MIDAS models

Consider the generic MIDAS model, written for quarterly, monthly and daily regressors, as:

$$q_t^h = G(\boldsymbol{\kappa}, \mathbf{x}_{t-h}) + \varepsilon_t^h$$

where:

$$\mathbf{x}_{t-h} = \left(y_{t-h}, \dots, y_{t-h-p+1}, x_{t-(3h/m^M)+l^M}^M, \dots, x_{t-(3h/m^M)-(p^M-1/m^M)+l^M}^M, x_{t-(60h/m^D)+l^D}^D, \dots, x_{t-(60h/m^D)-l^D}^D \right)$$

and $\boldsymbol{\kappa} = (\theta^Q, \theta^M, \theta^D, \beta_Q, \beta_M, \beta_D)$.

Then the forecast error is given by:

$$e_{n|n-h} = q_n - q_{n|n-h}$$

where $q_{n|n-h}$ is the forecast using the estimated model with observations up to $n-h$, so the forecast error is:

$$e_{n|n-h} = \varepsilon_n^h + G(\boldsymbol{\kappa}, \mathbf{x}_{n-h}) - G(\hat{\boldsymbol{\kappa}}, \mathbf{x}_{n-h}) \quad (15)$$

Using a Taylor series expansion when $\hat{\boldsymbol{\kappa}}$ is close to $\boldsymbol{\kappa}$ (as will be the case for a reasonable sample size):

$$G(\hat{\boldsymbol{\kappa}}, \mathbf{x}_{t-h}) \approx G(\boldsymbol{\kappa}, \mathbf{x}_{t-h}) + \frac{\partial G(\boldsymbol{\kappa}, \mathbf{x}_{t-h})}{\partial \hat{\boldsymbol{\kappa}}} (\hat{\boldsymbol{\kappa}} - \boldsymbol{\kappa}) \quad (16)$$

$$= G(\boldsymbol{\kappa}, \mathbf{x}_{t-h}) + \mathbf{x}_{t-h}(\hat{\boldsymbol{\kappa}}) (\hat{\boldsymbol{\kappa}} - \boldsymbol{\kappa}) \quad (17)$$

Substituting from (16) into (15) gives:

$$e_{n|n-h} = \varepsilon_n^h + \mathbf{x}_{n-h}(\hat{\boldsymbol{\kappa}}) (\hat{\boldsymbol{\kappa}} - \boldsymbol{\kappa})$$

so that

$$\text{var}(e_{n|n-h}) = \sigma_{\varepsilon, n-h}^2 + \mathbf{x}_{n-h}(\hat{\boldsymbol{\kappa}}) \text{var}(\hat{\boldsymbol{\kappa}} - \boldsymbol{\kappa}) \mathbf{x}_{n-h}(\hat{\boldsymbol{\kappa}})' \quad (18)$$

So assuming an estimator $\widehat{\text{var}}(\hat{\boldsymbol{\kappa}})$, we can calculate the second term in the above expression, which is the contribution of parameter estimation uncertainty to the model's *ex ante* uncertainty. Write the full sample ($t = 1, 2, \dots, n-h$) gradient as $\mathbf{x}(\hat{\boldsymbol{\kappa}}) = \partial G(\hat{\boldsymbol{\kappa}}, \mathbf{x}) / \partial \hat{\boldsymbol{\kappa}}$. Then $\widehat{\text{var}}(\hat{\boldsymbol{\kappa}})$ is computed using a sandwich variance-covariance matrix by applying the usual Newey-West formulae to the full sample gradient $\mathbf{x}(\hat{\boldsymbol{\kappa}})$ and the residuals $\hat{\boldsymbol{\varepsilon}} = \mathbf{q}^h - G(\hat{\boldsymbol{\kappa}}, \mathbf{x}_{t-h})$. We compute the required gradients using numerical derivatives.

References

- Aiolfi, M., Capistrán, C., and Timmermann, A. (2011). Forecast combinations, chapter 11. In Clements, M. P., and Hendry, D. F. (eds.), *The Oxford Handbook of Economic Forecasting*, pp. 355–388: Oxford University Press.
- Andreou, E., Ghysels, E., and Kourtellos, A. (2010). Should macroeconomic forecasters use daily financial data and how?. Working paper, University of North Carolina.
- Andreou, E., Ghysels, E., and Kourtellos, A. (2011). Forecasting with mixed-frequency data, chapter 8. In Clements, M. P., and Hendry, D. F. (eds.), *The Oxford Handbook of Economic Forecasting*, pp. 225–246: Oxford University Press.
- Ang, A., Bekaert, G., and Wei, M. (2007). Do macro variables, asset markets, or surveys forecast inflation better?. *Journal of Monetary Economics*, **54**(4), 1163–1212.
- Baillie, R. T., and Bollerslev, T. (1992). Prediction in dynamic models with time-dependent conditional variances. *Journal of Econometrics*, **52**, 91–113.
- Bloom, N. (2009). The impact of uncertainty shocks. *Econometrica*, **77**, 623–685.
- Bloom, N. (2013). Fluctuations in uncertainty. National Bureau of Economic Research Working Paper 19714.
- Boero, G., Smith, J., and Wallis, K. F. (2012). The measurement and characteristics of professional forecasters’ uncertainty. Discussion paper, Department of Economics, University of Warwick.
- Carriero, A., Clark, T. E., and Marcellino, M. (2014). Real-time nowcasting with Bayesian mixed frequency model with stochastic volatility. Mimeo, Queen Mary, University of London.
- Carroll, C. D. (1996). Buffer-stock saving and the life cycle/permanent income hypothesis. Nber working paper no. 5788.
- Clements, M. P. (2014). Forecast Uncertainty - Ex Ante and Ex Post: US Inflation and Output Growth. *Journal of Business & Economic Statistics*, **32**(1). DOI: 10.1080/07350015.2013.859618.
- Clements, M. P., and Galvão, A. B. (2008). Macroeconomic forecasting with mixed-frequency data: Forecasting output growth in the United States. *Journal of Business and Economic Statistics*, **26**, 546–554. No. 4.
- Clements, M. P., and Galvão, A. B. (2013). Anticipating early data revisions to US output growth. Mimeo, Warwick University.
- Cogley, T., Sargent, T. J., and Surico, P. (2012). Price-level uncertainty and stability in the UK. mimeo, New York University.

- Croushore, D. (1993). Introducing: The Survey of Professional Forecasters. *Federal Reserve Bank of Philadelphia Business Review*, "November", 3–13.
- Croushore, D., and Stark, T. (2001). A real-time data set for macroeconomists. *Journal of Econometrics*, **105**(1), 111–130.
- Dixit, A., and Pindyck, R. (1994). *Investment under uncertainty*. Princeton: Princeton University Press.
- ECB (2009). Results of a special questionnaire for participants in the ECB survey of professional forecasters. Available at: http://www.ecb.europa.eu/stats/prices/indic/forecast/shared/files/quest_summary.pdf, mimeo, European Central Bank, Eurosystem.
- Faust, J., and Wright, J. H. (2009). Comparing Greenbook and reduced form forecasts using a large realtime dataset. *Journal of Business and Economic Statistics*, **27** (4), 468–479.
- Faust, J., and Wright, J. H. (2012). Forecasting Inflation. Discussion paper, Department of Economics, John Hopkins University.
- Garcia, J. A. (2003). An introduction to the ECB's Survey of Professional Forecasters. Discussion paper, Occasional Paper Series No. 8, European Central Bank.
- Ghysels, E., Santa-Clara, P., and Valkanov, R. (2006). Predicting volatility: Getting the most out of return data sampled at different frequencies. *Journal of Econometrics*, **131**, 59–95.
- Ghysels, E., Sinko, A., and Valkanov, R. (2007). MIDAS regressions: Further results and new directions. *Econometric Reviews*, **26**, 53–90.
- Ghysels, E., and Wright, J. (2009). Forecasting Professional Forecasters. *Journal of Business and Economic Statistics*, **27**, 504–516.
- Gilbert, T., Scotti, C., Strasser, G., and Vega, C. (2010). Why do certain macroeconomic news have a big impact on asset prices. mimeo.
- Giordani, P., and Söderlind, P. (2003). Inflation forecast uncertainty. *European Economic Review*, **47**(6), 1037–1059.
- Hansen, B. E. (1997). Approximate asymptotic p values for structural-change tests. *Journal of Business & Economic Statistics*, *15*(1), 60–67.
- Herbst, E., and Schorfheide, F. (2012). Evaluating DSGE model forecasts of comovements. *Journal of Econometrics*, **171**, 152–166.
- Hirshleifer, D. (2001). Investor psychology and asset pricing. *Journal of Finance*, **56**(4), 1533–1597.
- Jurado, K., Ludvigson, S. C., and Ng, S. (2013). Measuring Uncertainty. mimeo, Depart-

ment of Economics, Columbia University.

- McConnell, M. M., and Perez-Quiros, G. (2000). Output fluctuations in the United States: What has changed since the early 1980s?. *American Economic Review*, **90**, 1464–1476.
- O’Hagan, A., Buck, C. E., Daneshkhah, A., Eiser, J. R., Garthwaite, P. H., Jenkinson, D. J., Oakley, J. E., and Rakow, T. (2006). *Uncertain Judgements: Eliciting Experts’ Probabilities*: John Wiley and Sons, Ltd.
- Rabin, M. (1998). Psychology and economics. *Journal of Economic Literature*, **36(1)**, 11–46.
- Sensier, M., and van Dijk, D. (2004). Testing for volatility changes in u.s. macroeconomic time series. *The Review of Economics and Statistics*, *86(3)*, 833–839.
- Stark, T. (2013). SPF Panelists’ Forecasting Methods: A note of the aggregate results of a November 2009 Special Survey. Available at: <http://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/>, Research Department, Federal Reserve Bank of Philadelphia.
- Stock, J. H., and Watson, M. W. (2007). Why has U.S. Inflation Become Harder to Forecast?. *Journal of Money, Credit and Banking*, **Supplement to Vol. 39**, 3–33.
- Stock, J. H., and Watson, M. W. (2008). Phillips Curve Inflation Forecasts. Working paper 14322, NBER, Cambridge, MA.
- Zarnowitz, V. (1969). The new ASA-NBER Survey of Forecasts by Economic Statisticians. *The American Statistician*, **23, No. 1**, 12–16.
- Zarnowitz, V., and Braun, P. (1993). Twenty-two years of the NBER-ASA quarterly economic outlook surveys: aspects and comparisons of forecasting performance. In Stock, J., and Watson, M. (eds.), *Business Cycles, Indicators, and Forecasting*, pp. 11–84: Chicago: University of Chicago Press and NBER.
- Zarnowitz, V., and Lambros, L. A. (1987). Consensus and uncertainty in economic prediction. *Journal of Political Economy*, **95(3)**, 591–621.

Figure 1: Term Structure of Survey Forecasting Uncertainties.

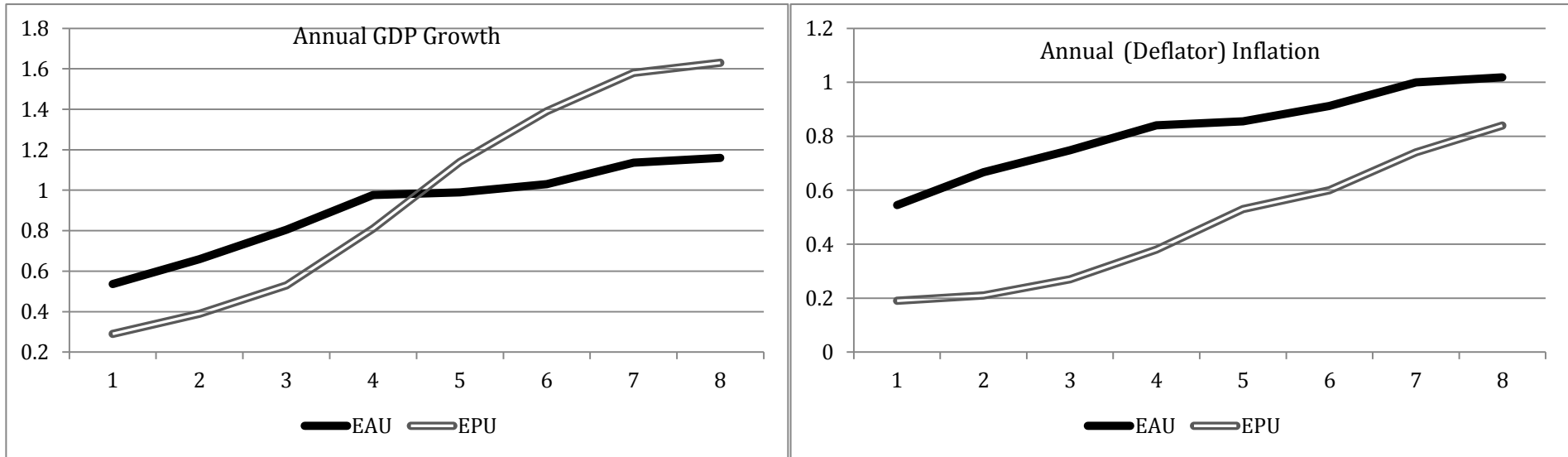


Figure 2: Term Structure of UCSV Forecasting Uncertainties of Inflation.

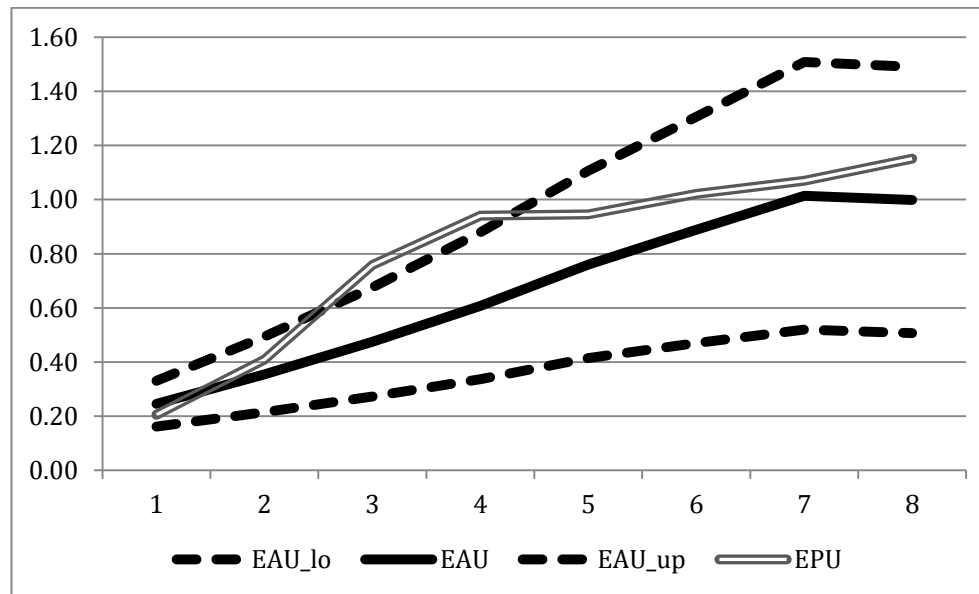


Figure 3: Term Structure of MIDAS Forecasting Uncertainties. (MIDAS_FD: dot. MIDAS_MD: max/min over 10 specifications.)

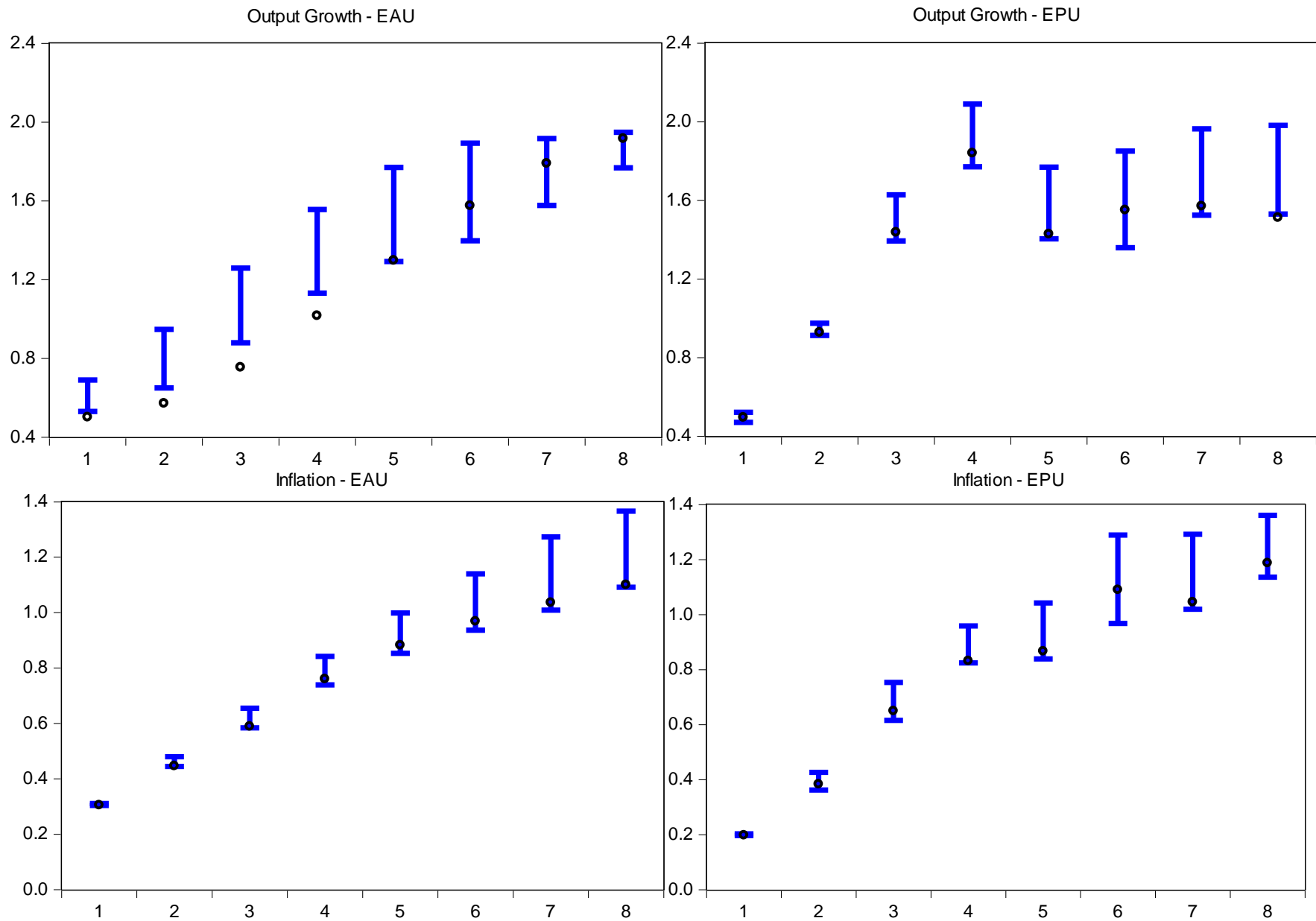


Figure 4: Time Variation of Survey and Model Ex ante Uncertainties (EAU).

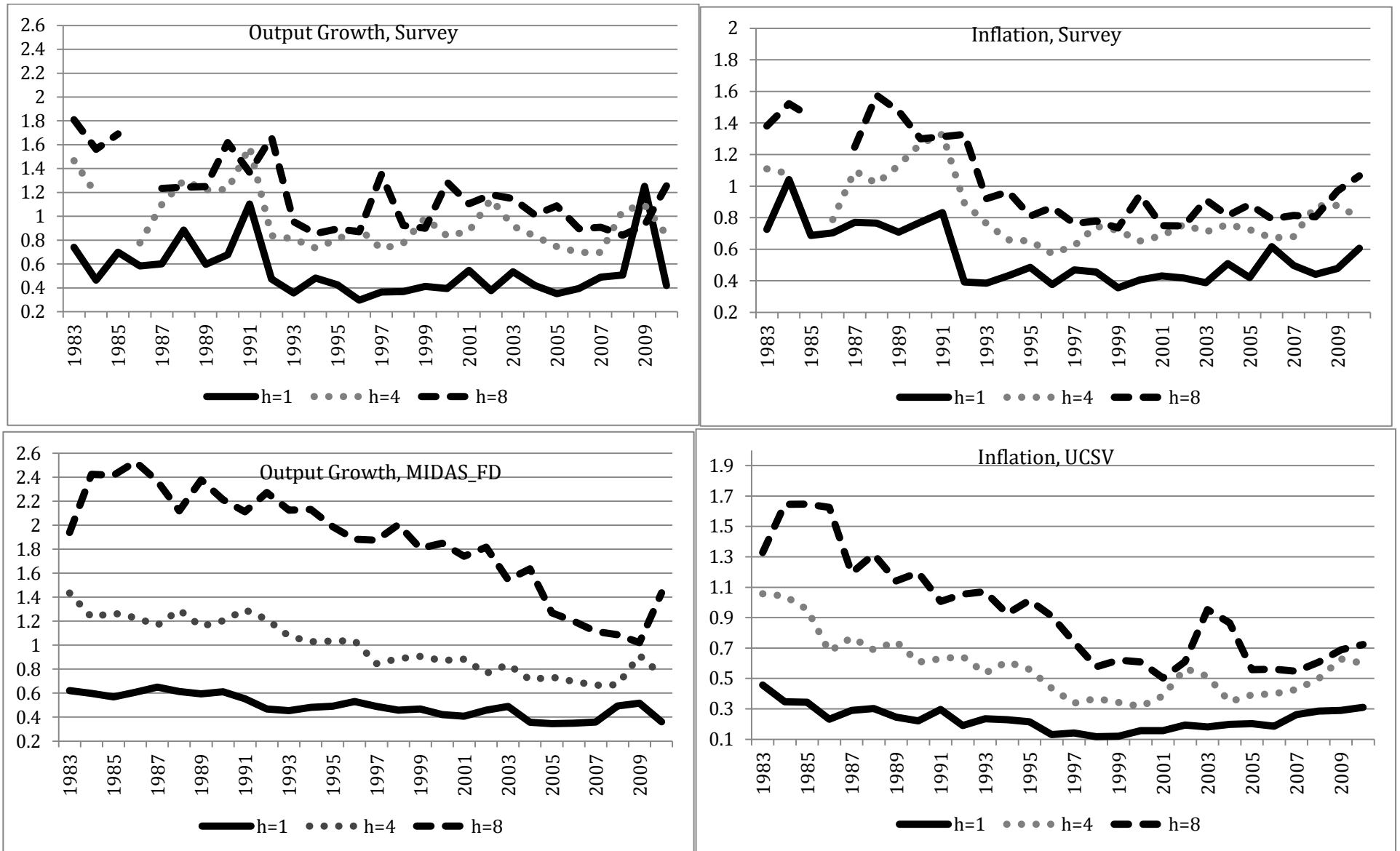


Figure 5: Recursive, Rolling and After-the-break Recursive EAU estimates with the MIDAS_FD Model.

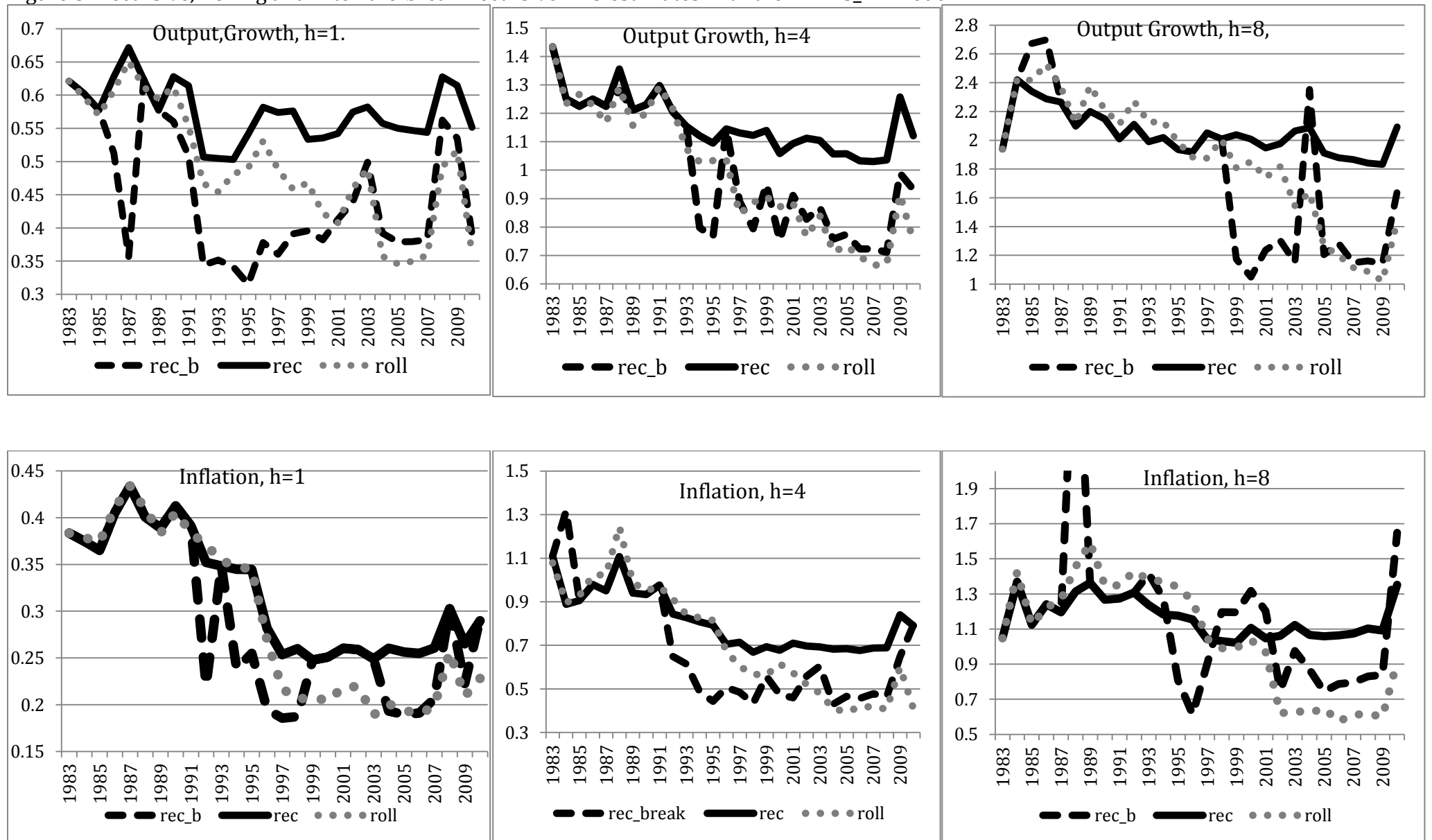


Figure 6: Time Variation of EAU: Survey and Models for h=1, 4, 8 .

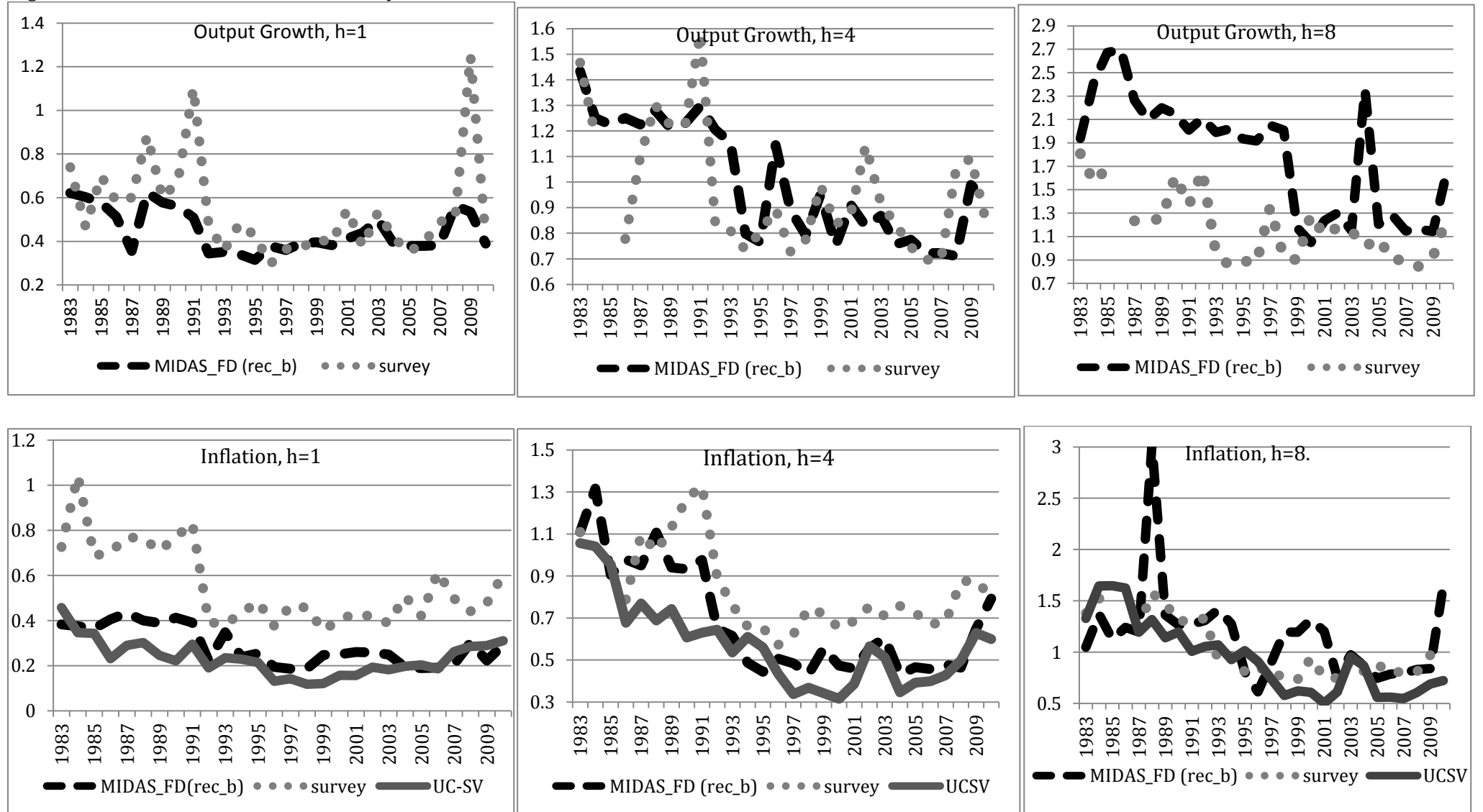


Figure 7: Predicted Daily Volatility (in basis points) and Model (roll +vol) EAU for h=1, 4 and 8.

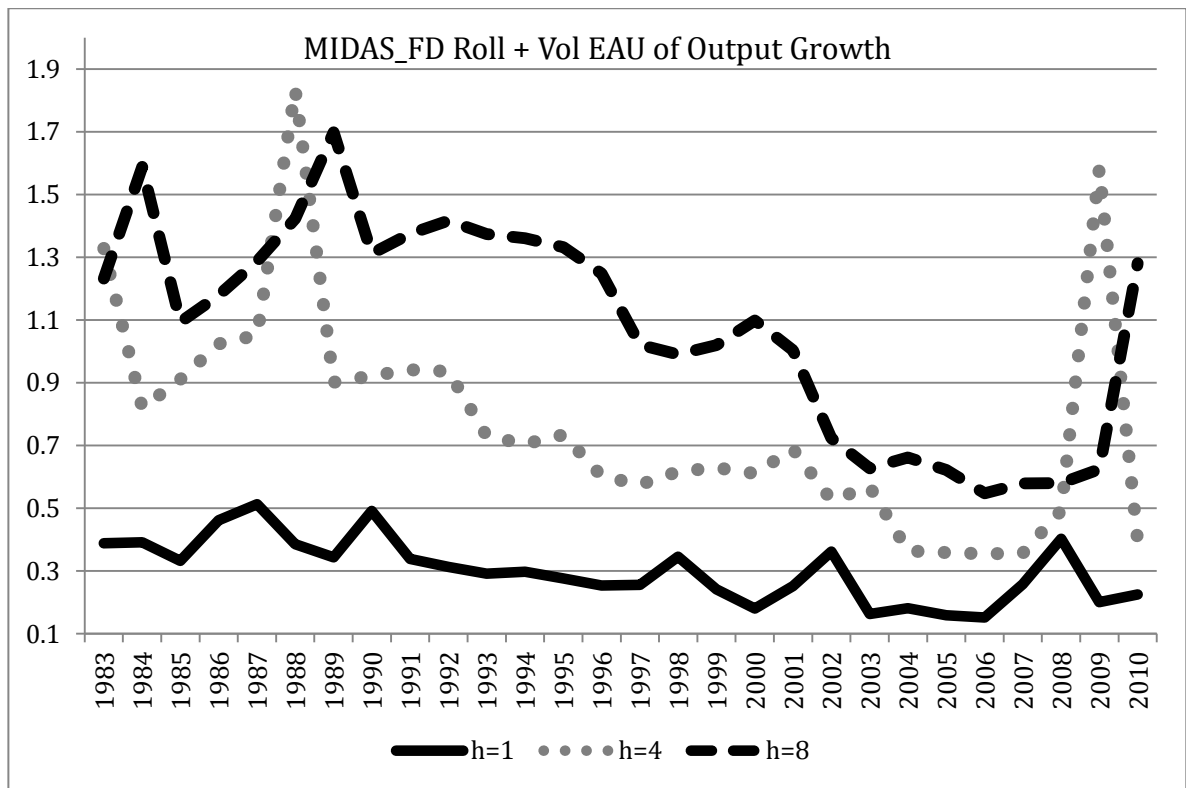
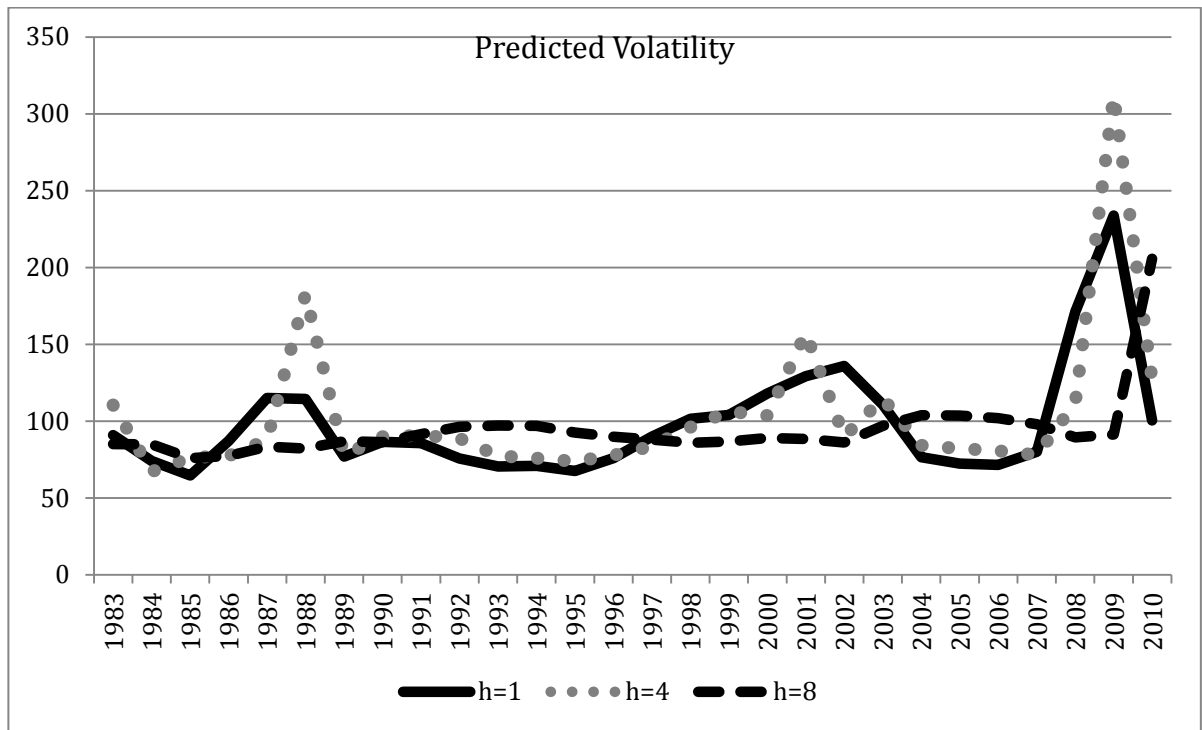


Table 1: Monthly Indicators

Variable	Description	Vintages Available	Source
Ind. Prod.	Total Industrial Production	1966:M1-2011:M6	RTDSM – Philadelphia Fed
Empl.	Employees on non-agricultural payrolls	1966:M1-2011:M6	RTDSM – Philadelphia Fed
Sales	Retail and Food Services Sales; Retail Sales before vintage 1992:M1.	1966:M1-2011:M6	ALFRED – St Louis Fed.
Housing	New Privately Owned Houses Started	1968:M2-2011:M6;	RTDSM – Philadelphia Fed
PMI	Production Manufacturing Index: ISM since 2002, but previously NAPM.	Obs: 1959:M1-2011-M6 (not subject to revisions)	ALFRED – St Louis Fed

Table 2: Correlation between Survey EAU and Model-based EAU measures (1983-2010)

Table 2A: Output Growth

	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8
MIDAS_FD: roll	0.49	0.57	0.82	0.63	0.69	0.65	0.59	0.55
MIDAS_FD: rec	0.53	0.64	0.78	0.76	0.75	0.76	0.61	0.60
MIDAS_FD: rec_b	0.60	0.70	0.76	0.68	0.78	0.74	0.49	0.49
MIDAS_Vol:	0.48	0.52	0.21	0.22	0.32	-0.18	-0.36	-0.09
MIDAS_FD: roll+vol	0.61	0.71	0.67	0.66	0.60	0.62	0.56	0.58

Table 2B: Inflation

	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8
UCSV	0.66	0.66	0.80	0.68	0.80	0.82	0.78	0.85
MIDAS_FD: roll	0.66	0.70	0.57	0.64	0.75	0.71	0.66	0.64
MIDAS_FD: rec	0.73	0.77	0.68	0.76	0.88	0.87	0.80	0.73
MIDAS_FD: rec_b	0.76	0.71	0.69	0.80	0.87	0.83	0.69	0.59
MIDAS_Vol:	-0.19	-0.06	0.02	0.07	0.22	-0.25	-0.37	-0.12
MIDAS_FD: roll+vol	0.64	0.65	0.49	0.59	0.59	0.68	0.64	0.65