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Should Portfolio Model Inputs Be Estimated Using One or Two Economic Regimes?

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Abstract
Estimation errors in the inputs are the main problem when applying portfolio analysis. Markov regime switching models are used to reduce these errors, but they do not always improve out-of-sample portfolio performance. We investigate the levels of transaction costs and risk aversion below which the use of two regimes is superior to one regime for an investor with a CRRA utility function, allowing for skewed and kurtic returns. Our results suggest that, due to differences in risk and transactions costs, most retail investors should use one regime models, while investment banks should use two regime models.

Keywords
finance, portfolio theory, regime shifting, transaction costs, risk aversion, constant relative risk aversion

JEL Classifications
G11

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Introduction

Estimation errors in the input parameters are the major problem when using portfolio theory (Markowitz, 1952). The means, variances and correlations of asset returns are usually estimated by their historical values, or their transformations. But the performance of portfolio models with inputs estimated in this way has been disappointing due to large estimation errors (Benati, 2015; Dhingra, 1983; Hui, et al., 1993; Levy and Simaan, 2016; Maillet, et al., 2015). In recent years a new approach to estimating portfolio inputs has been developed which recognises that asset returns are not generated by a single economic regime (Levy and Kaplanski, 2015; Bae, et al., 2014). The economy and financial markets switch between regimes, e.g. expansion and recession, bull and bear markets; and the portfolio input parameters vary with the regime. Therefore a different set of parameters needs to be estimated for each economic regime. This is done using Markov regime switching models which allow the various economic regimes to be identified, the input parameters for each regime to be estimated, and the regime applying next period to be forecast. Investors have to choose between using one and multi-regime models when estimating the portfolio input parameters. To help investors make this choice, we compare the out-of-sample performance of portfolios formed using both one and two regimes.

Three papers have compared the out-of-sample performance of one and two or more regimes when short sales are banned (Angelidis & Tessaromatis, 2014; Guidolin & Timmermann, 2007; and Guidolin & Ria, 2011). Asset returns are usually skewed and display kurtosis, but only Guidolin & Timmermann (2007) have formed portfolios by maximising CRRA utility, which allows for skewness and kurtosis. We conduct the first study of the effects of varying transaction costs and risk aversion on the choice between forming portfolios using one and two regimes.

Data and Methodology

We analyse monthly returns in US dollars from July 1961 to December 2015 on an index of US equities (value-weighted total returns for all CRSP firms incorporated in the US and listed on NYSE, AMEX and NASDAQ), and an index of US nominal bonds (US Treasury 10-year bond total returns). Following Board and Sutcliffe (1994), DeMiguel et al. (2009), Tu and Zhou (2011) and Platanakis and Sutcliffe (2017), amongst others; our analysis is based on a 'rolling-window' approach. Specifically, we consider a rolling estimation window of 120 months (10 years) and use the data in each estimation period to compute the optimal asset allocation for each portfolio technique over the next out-of-sample month. Then we roll the estimation window forward by 1 month and resolve the optimization problem; repeating this process until we reach our data horizon.

It has been widely reported in the literature that portfolio construction techniques based only on the first two statistical moments (mean-variance approaches) suffer when asset returns do not follow a normal distribution, see for instance Cumming et al. (2014) and Xiong and Idzorek (2011), amongst others. We use a Taylor series expansion for the CRRA (Constant Relative Risk Aversion) utility function in order to incorporate higher moments in the portfolio construction process. CRRA utility is given by:

$$U_{CRRA}\left(W\right)=\frac{1}{1-\lambda}W^{1-\lambda},\ \text{if } \lambda>0,\ \lambda\neq 1,$$

where W denotes the end-of-period wealth, and λ is the risk aversion parameter. Following Benishay (1992), Jondeau and Rockinger (2006), and many others, we use a Taylor series expansion up to the 4th moment to approximate the expected CRRA utility. The Taylor series expansion for expected utility up to the 4th order and centered at $\bar{W}$ is given by:

$$E\left(U\left(W\right)\right)\approx U\left(\bar{W}\right)+U'\left(\bar{W}\right)E\left[W-\bar{W}\right]+\frac{1}{2}U''\left(\bar{W}\right)E\left[\left(W-\bar{W}\right)^{2}\right]+$$
$$+\frac{1}{3!}U'''\left(\bar{W}\right)E\left[\left(W-\bar{W}\right)^{3}\right]+\frac{1}{4!}U''''\left(\bar{W}\right)E\left[\left(W-\bar{W}\right)^{4}\right]+O\left(W^{4}\right)$$

where $O\left(W^{4}\right)$ denotes the Taylor series remainder. As a result, the Taylor series expansion of expected CRRA utility up to the 4th order (centered at $\bar{W}$) is given by:

$$E\left(U_{CRRA}\left(W\right)\right)\approx\frac{1}{1-\lambda}\bar{W}^{1-\lambda}-\frac{\lambda}{2}\bar{W}^{-(\lambda+1)}\sigma_{\mathrm{p}}^{2}+\frac{\lambda\left(\lambda+1\right)}{3!}\bar{W}^{-(\lambda+2)}\mathrm{s}_{\mathrm{p}}^{3}-\frac{\lambda\left(\lambda+1\right)\left(\lambda+2\right)}{4!}\bar{W}^{-(\lambda+3)}\mathrm{k}_{\mathrm{p}}^{4}$$

where $\bar{W}=1+\mu_{\mathrm{p}}$ denotes the expected end-of-period wealth. The statistical moments $\mu_{\mathrm{p}},\ \sigma_{\mathrm{p}}^{2},\ \mathrm{s}_{\mathrm{p}}^{3},\ \mathrm{k}_{\mathrm{p}}^{4}$ represent respectively the expected return, variance, skewness and kurtosis of the portfolio return using sample-based estimates for a given vector of asset weights, and are defined as follows:

$$\mu_{\mathrm{p}} = \mathbf{x}^{\mathrm{T}}\boldsymbol{\mu}$$

$$\sigma_{\mathrm{p}}^{2} = \mathbf{x}^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{x}$$

$$\mathrm{s}_{\mathrm{p}}^{3} = \mathbf{x}^{\mathrm{T}}\mathbf{M}_{3}\left(\mathbf{x}\otimes\mathbf{x}\right)$$

$$\mathrm{k}_{\mathrm{p}}^{4} = \mathbf{x}^{\mathrm{T}}\mathbf{M}_{4}\left(\mathbf{x}\otimes\mathbf{x}\otimes\mathbf{x}\right)$$

where $\mathbf{M}_3$ and $\mathbf{M}_4$ represent the co-skewness and co-kurtosis matrices respectively, and the symbol $\otimes$ stands for the Kronecker product. We maximize the expected CRRA utility function in terms of the portfolio weights subject to no short selling constraints and normalization of portfolio weights (e.g. they sum to 1).

Our model subtracts transaction costs (allowing for an expected holding period of 21 months) from returns in the objective function, and when computing out-of-sample returns. We use 50 bps as the transaction costs of US equities (DeMiguel et al, 2009), and 17 bps for bonds (Edwards et al, 2007). We vary risk aversion from 2 to 10, and transaction costs of zero, one and two times their estimated level.

We estimate a two state multi-variate regime-switching model for each rolling estimation period:

$$\mathbf{y}_t = \boldsymbol{\mu}_1 + \boldsymbol{\varepsilon}_{t1},\ \ \boldsymbol{\varepsilon}_{t1} \sim MVN\left(0,\ \boldsymbol{\Sigma}_1\right) \quad \text{for state 1 } \left(\mathrm{S}_t = 1\right)$$

$$\mathbf{y}_t = \boldsymbol{\mu}_2 + \boldsymbol{\varepsilon}_{t2},\ \ \boldsymbol{\varepsilon}_{t2} \sim MVN\left(0,\ \boldsymbol{\Sigma}_2\right) \quad \text{for state 2 } \left(\mathrm{S}_t = 2\right)$$

where $\mathbf{y}_t$ is a 2×1 vector of asset returns, $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ are 2×1 vectors of mean asset returns for states 1 and 2 respectively. $\boldsymbol{\varepsilon}_{t1}$ and $\boldsymbol{\varepsilon}_{t2}$ follow a multivariate normal distribution with zero mean and a 2×2 covariance matrix given by $\boldsymbol{\Sigma}_1$ and $\boldsymbol{\Sigma}_2$ for states 1 and 2. We assume a Markov process with a $\left(2\times2\right)$ transition matrix $\boldsymbol{\Pi}$ characterised by constant probabilities $(p,\ q)$ defined as:

$$\boldsymbol{\Pi} = \begin{bmatrix} \Pr\left(S_t = 1,\ S_{t-1} = 1\right) & \Pr\left(S_t = 1,\ S_{t-1} = 2\right) \\ \Pr\left(S_t = 2,\ S_{t-1} = 1\right) & \Pr\left(S_t = 2,\ S_{t-1} = 2\right) \end{bmatrix} = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}$$

At any given month t, $\mathbf{y}_t$ follows the distribution associated with state S at month t. If at month $t+1$ $\mathbf{y}_{t+1}$ remains in the same regime, it follows the distribution at the given transition probability, p or q; if at month $t+1$ $\mathbf{y}_{t+1}$ switches to the other regime, it follows the distribution of the other regime at the given transition probability, $1-p$ or $1-q$. For each month, when the smoothed conditional probability over the estimation period is higher (lower) than 50%, that month is classified as being in state 1 (state 2), Kim (1994). The smoothed probability is the probability of being in state 1 or state 2 conditional over the sample period. Following Ang and Bekaert (2004), when the realisation of the regime in the last month of each period is in state 1 (S = 1), π_1 equals the transition probability p, i.e. $\pi_1 = p$. When the realisation of the regime in the last month of each period is in state 2 (S = 2), π_1 equals the transition probability $1-q$, i.e. $\pi_1 = 1-q$.

Based on the realization of the regime in the last month of each estimation period, we compute the first four moments and the covariance matrix, as defined by Timmermann (2000), for the next out-of-sample month. The first four moments are defined as:

$$\mu_{\mathrm{p}} = \pi_1 \mu_1^{\mathbf{x}_1} + (1-\pi_1)\mu_2^{\mathbf{x}_2}$$

$$\sigma_{\mathrm{p}}^2 = \pi_1(1-\pi_1)\left(\mu_1^{\mathbf{x}_1} - \mu_2^{\mathbf{x}_2}\right)^2 + \pi_1 \sigma_1^{\mathbf{x}_1 2} + (1-\pi_1)\sigma_2^{\mathbf{x}_2 2}$$

$$s_{\mathrm{p}}^3 = \pi_1(1-\pi_1)\left(\mu_1^{\mathbf{x}_1} - \mu_2^{\mathbf{x}_2}\right)\left[(1-2\pi_1)\left(\mu_1^{\mathbf{x}_1} - \mu_2^{\mathbf{x}_2}\right)^2 + 3\left(\sigma_1^{\mathbf{x}_1 2} - \sigma_2^{\mathbf{x}_2 2}\right)\right]$$

$$k_{\mathrm{p}}^4 = \pi_1(1-\pi_1)\left(\mu_1^{\mathbf{x}_1} - \mu_2^{\mathbf{x}_2}\right)^2\left[\left((1-\pi_1)^3 + \pi_1^3\right)\left(\mu_1^{\mathbf{x}_1} - \mu_2^{\mathbf{x}_2}\right)^2 + 6\pi_1 \sigma_2^{\mathbf{x}_2 2} + 6(1-\pi_1)\sigma_1^{\mathbf{x}_1 2}\right] + 3\pi_1 \sigma_1^{\mathbf{x}_1 4} + 3(1-\pi_1)\sigma_2^{\mathbf{x}_2 4}$$

where $\mu_1^{\mathbf{x}_1}$ and $\mu_2^{\mathbf{x}_2}$ are the mean portfolio returns for the states 1 and 2, defined as $\mu_1^{\mathbf{x}} = \mathbf{x}_1{}^{\mathrm{T}} \boldsymbol{\mu}_1$ and $\mu_2^{\mathbf{x}} = \mathbf{x}_2{}^{\mathrm{T}} \boldsymbol{\mu}_2$; $\sigma_1^{\mathbf{x}_1 2}$ and $\sigma_2^{\mathbf{x}_2 2}$ are the variances of the portfolio returns for the states 1 and 2, defined as $\sigma_1^{\mathbf{x}_1 2} = \mathbf{x}_1{}^{\mathrm{T}} \boldsymbol{\Sigma}_1 \mathbf{x}_1$ and $\sigma_2^{\mathbf{x} 2} = \mathbf{x}_2{}^{\mathrm{T}} \boldsymbol{\Sigma}_2 \mathbf{x}_2$; $\mathbf{x}_1$ and $\mathbf{x}_2$ are vectors of portfolio weights for the states 1 and 2, respectively. We then use these estimates as inputs to our portfolio model.

The value of the regime classification measure (RCM) of Ang and Bekaert (2002a) is 28.00, supporting the validity of our classification of the data into two regimes. Descriptive statistics for one and two regimes for the full out-of-sample period appear in Table 1. Ang and Bekaert (2002b) find that in the high volatility regime equities have lower mean returns, while Ang and

Bekaert (2002a) find that in the high volatility regime bonds have higher returns. Table 1 shows these conclusions also apply to our data.

Table 1. Descriptive Statistics for One and Two Regimes for the Entire Out-of-Sample Period – Annualized Returns

No. of Regimes	Asset	Mean	Std. Dev.	Skewness	Kurtosis
One Regime	Equities	10.639%	15.374%	–0.142	3.156
	Bonds	6.749%	6.062%	0.106	3.357
Two Regimes – Low Volatility	Equities	14.050%	11.140%	–0.056	3.028
	Bonds	3.764%	3.420%	–0.031	3.022
Two Regimes – High Volatility	Equities	5.998%	19.672%	–0.106	3.061
	Bonds	10.812%	8.283%	–0.001	3.143

Results

The six figures show the two regime scores, less those for one regime, where Figures 1 to 4 use annualized returns. Figures 1 and 2 illustrate the difference in performance between one and two regimes, as measured by certainly equivalent returns (CERs) and Sharpe ratios respectively. Broadly similar results are obtained using the Sortino, Dowd, Sterling Calmar and Omega measures, see the Appendix. The CER for a CRRA utility function is computed as $CER_t = \{(1-\lambda)E[Ut]\}1/(1-\lambda)-1$ where $E[U]$ is the mean utility across our out-of-sample periods (Diris et al, 2015). Figures 1 and 2 indicate that, as risk aversion increases, the relative attractiveness of using two regimes decreases, i.e. the ΔCERs become negative. Figures 3 and 4 show this is primarily due to an increase in the relative risk (difference in the standard deviations) of two regime models, rather than any change in expected returns. Cumulative wealth gives similar returns to expected returns, and downside standard deviation, Var(99%), expected drawdown, and maximum drawdown give similar results to the standard deviation, see the Appendix.

The choice between one and two regimes is not influenced by the third and fourth moments. Differences in skewness between one and two regimes are small for all values of λ. Differences in kurtosis are also minimal when $\lambda = 2$ and $\lambda = 10$, although they are significant for some intermediate values of λ. Looking at the ΔCERs in Figure 1, when transaction costs are zero the use of regimes is preferable for investors with risk aversion below six. In Figure 2, the corresponding number for Sharpe ratios is almost nine. Figures 1, 2 and 4 indicate that higher transactions costs favour two regimes at all levels of risk aversion.

Figures 5 and 6 show that, irrespective of the levels of transaction costs and risk aversion, the use of two regimes generates less diverse and less stable portfolios than does one regime, leading to higher transactions costs. Diversification is measured as in equation (13) of Platanakis and Sutcliffe (2017), where full diversification scores *1/N* and zero diversification scores unity. Stability is the average value of the sum of squares of the differences between the portfolio proportion for each asset in adjacent time periods. As transactions costs increase, the ΔCER and ΔSharpe ratio plots in Figures 1 and 2 shift down, and the relative attractiveness of using two regimes decreases. When TC = 1 the level of risk aversion below which two regimes are preferable drops to 4.5 for CERs, and 6.0 for Sharpe ratios. For both CERs or Sharpe ratios, when transaction costs are double their estimated values the use of two regimes is never preferable.

Conclusions

When estimating the inputs required for portfolio analysis regime switching models have been used to reduce the estimation errors. Allowing for skewness and kurtosis and ruling out short sales, we have investigated the effects of transactions costs and risk aversion on the out-of-sample performance of regime shifting models. Highly risk averse investors with high transaction costs, who are likely to be retail investors (Dohmen, et al, 2005) should use one regime models. Investors with low risk aversion and low transaction costs, such as proprietary traders in investment banks (Kelly, 2010), should use two regimes.

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Figures and tables

Figure 1. Two Less One Regime ΔCERs

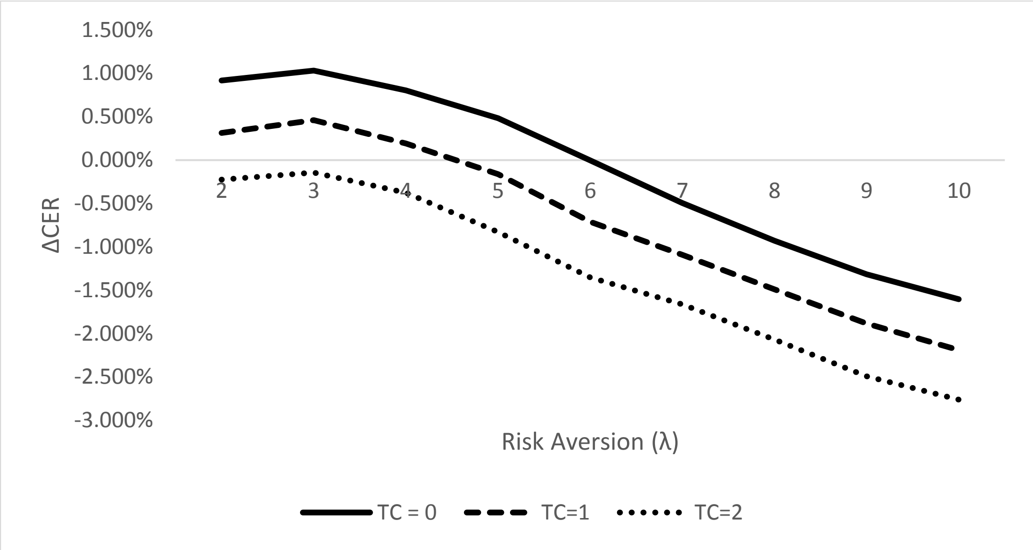


Figure 2. Two Less One Regime Sharpe Ratios

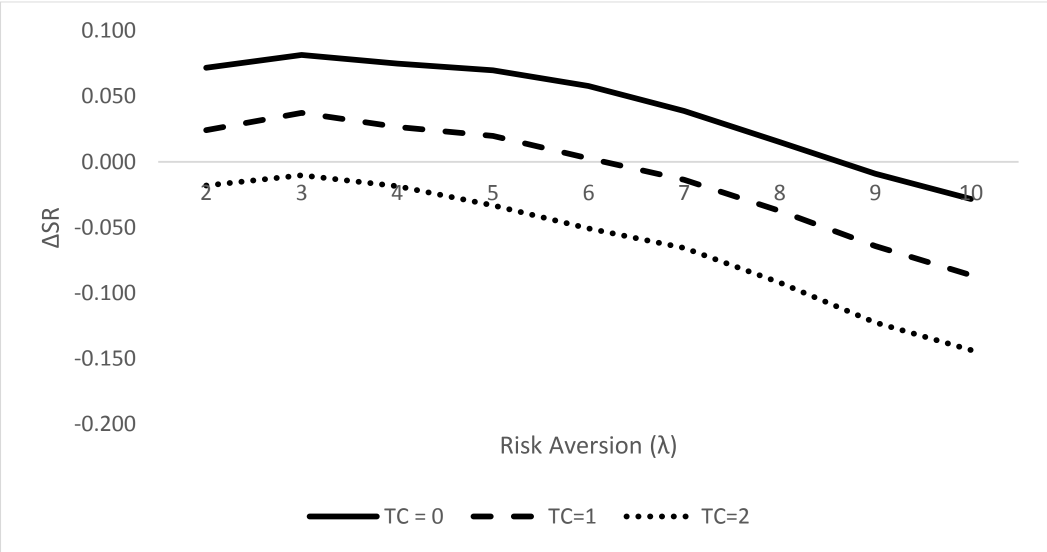


Figure 3. Two Less One Regime Standard Deviations

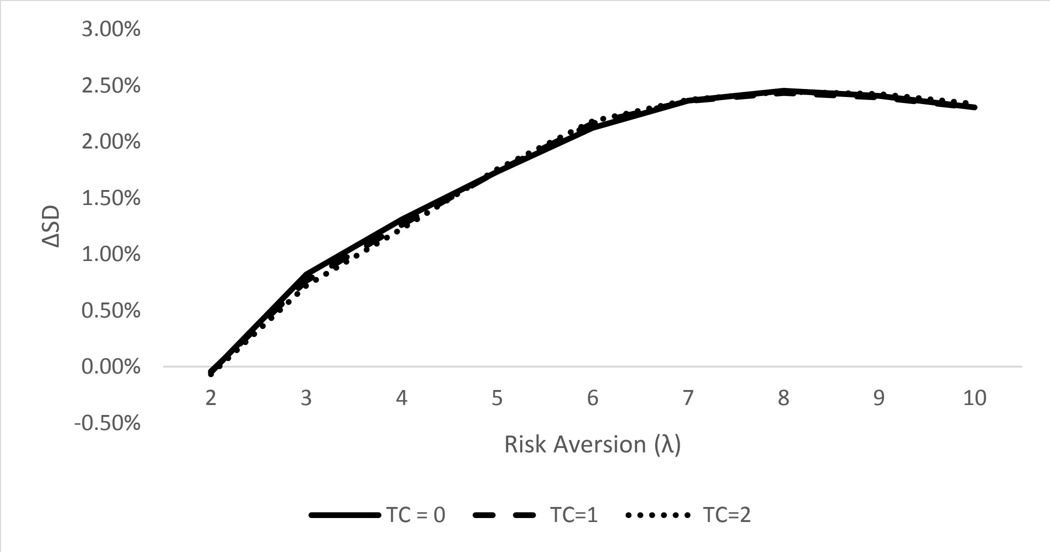


Figure 4. Two Less One Regime Expected Returns

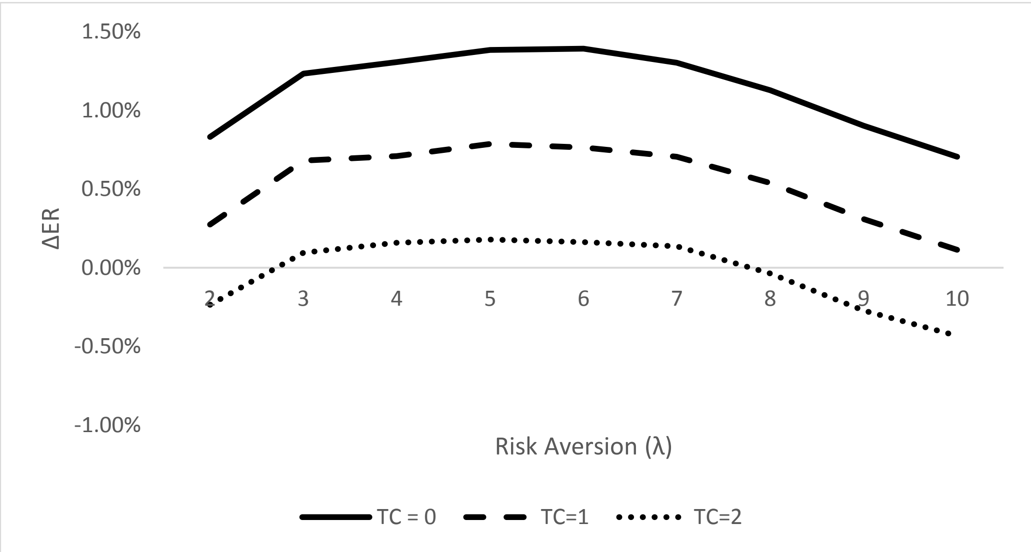


Figure 5. Two Less One Regime Diversification

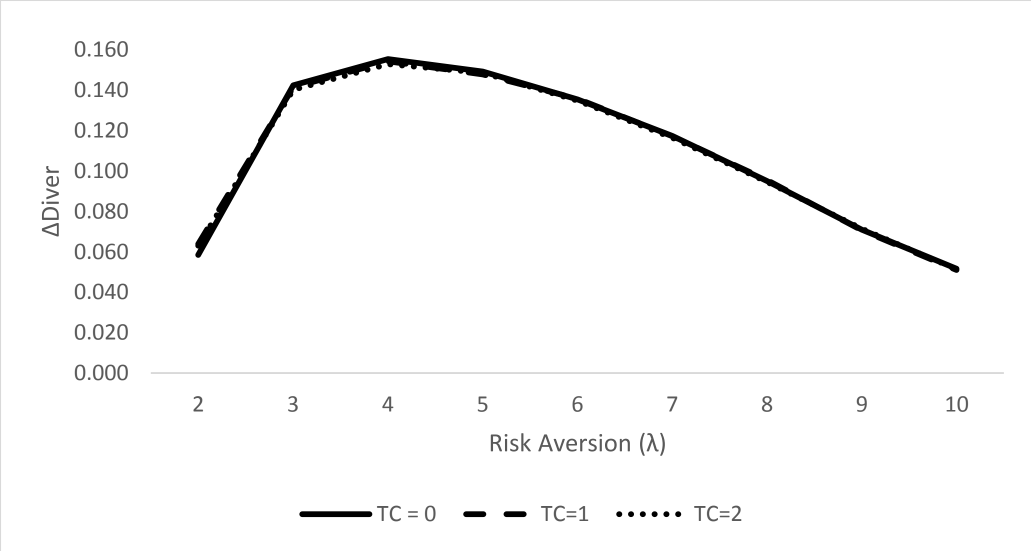
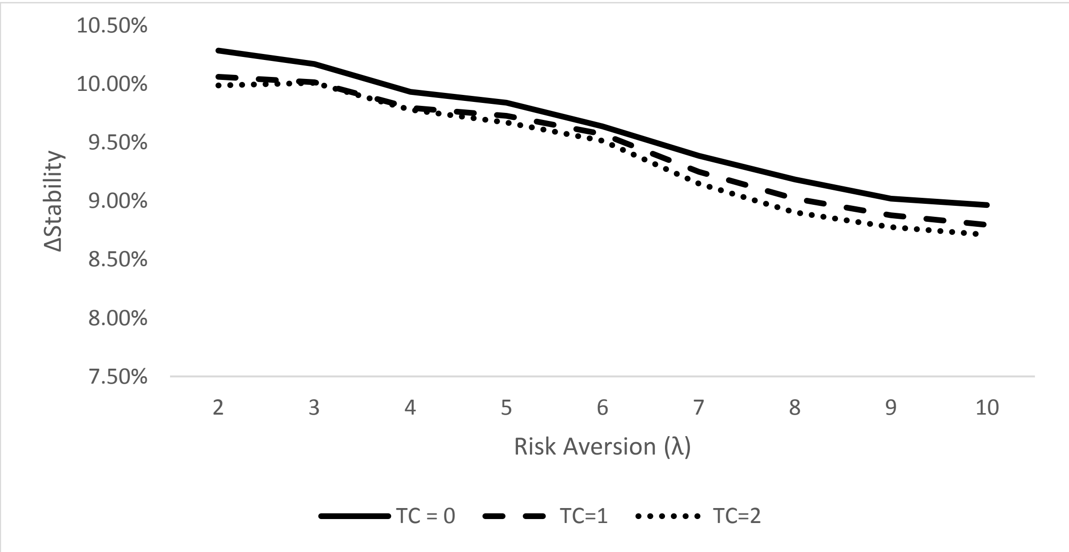


Figure 6. Two Less One Regime Stability



Appendix

Table 2: Annualized Score for Two Regimes Minus the Annualized Score for One Regime When Transactions Costs Equal One

Risk aversion (λ)	2	3	4	5	6	7	8	9	10
	Panel A. Risk Return Measures								
Sortino	0.063	0.084	0.067	0.050	0.009	–0.030	–0.086	–0.154	–0.210
Dowd	0.019	0.025	0.013	0.004	–0.015	–0.034	–0.059	–0.087	–0.111
Sterling	0.021	0.022	0.017	0.011	–0.003	–0.021	–0.043	–0.064	–0.082
Calmar	0.004	0.004	0.003	0.002	0.000	–0.002	–0.004	–0.006	–0.008
Omega	0.032	–0.018	–0.104	–0.174	–0.246	–0.318	–0.377	–0.441	–0.496
	Panel B. Returns								
Cumulative Wealth	4.900	9.155	8.225	8.271	7.221	6.281	4.043	1.220	–0.957
	Panel C. Risk								
Downside Std. Dev.	–0.003	0.002	0.005	0.009	0.012	0.014	0.015	0.016	0.016
VaR (99%)	–0.004	0.011	0.022	0.033	0.043	0.048	0.051	0.053	0.053
Expected Drawdown	–0.019	–0.013	–0.005	0.002	0.011	0.018	0.022	0.024	0.024
Max. Drawdown	–0.122	–0.081	–0.029	0.015	0.062	0.096	0.115	0.129	0.131

Results for TC = 0 and TC = 2 are available from the authors on request.