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Rounding Behaviour of Professional Macro-Forecasters

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Abstract

The rounding of point forecasts of CPI inflation and the unemployment rate by U.S. Professional Forecasters is modest. There is little evidence that forecasts are rounded to a greater extent in response to higher perceived uncertainty about future outcomes. There is clear evidence that probability of decline forecasts are rounded: over a half of the forecast probabilities of decline in the current quarter are multiples of 10. We find that rounding of these probabilities is correlated with worse accuracy, but are cognizant that worse (less accurate) forecasters might round more, rather than the degree of rounding *per se* worsening accuracy. By simulating the loss from rounding for a set of efficient forecasters, we show that the explanation that respondents round otherwise efficient forecasts is untenable, and that the contribution of rounding is of minor importance.

JEL: C53, D84

Rounding, survey expectations, uncertainty, forecast accuracy, histograms.

1 Introduction

In this paper we consider the extent to which *professional* forecasters round their forecasts, why they might round their forecasts, and the impact of this practice on forecast accuracy. A recent paper by Binder (2017) suggests that *consumers* round their inflation expectations to a striking degree when responding to surveys. Around a half of the respondents to the Michigan Survey of Consumers (MSC) round their forecasts of inflation one-year ahead to a multiple of 5%.[1] Perhaps not surprisingly, the respondents to the U.S. Survey of Professional Forecasters report point forecasts of inflation and other variables which are far less coarse, and evidence of rounding is less readily apparent. However, the SPF forecast probabilities of declines in GDP do portray clear evidence of rounding.

As to why respondents round their forecasts, Binder (2017) refers to the communication and linguistic theory literature, and the notion that *Round Numbers Suggest Round Interpretation* (or RNRI). That is, round numbers are used to convey uncertainty. She documents evidence in support of the RNRI principle in the finance literature, and in surveys of earnings and age, amongst other variables.[2] Using the Federal Reserve Bank of New York Survey of Consumer Expectations (SCE), which provides point predictions and density forecasts of inflation, Binder shows that higher inflation uncertainty, as measured by the inter-quartile range of an individual's inflation density forecast, is positively correlated with the rounding of the point forecast, supporting RNRI for consumers' inflation forecasts.[3] Manski and Molinari (2010) suppose survey responses might be rounded 'to simplify communication', or 'to convey ambiguity'. In response to the question 'How many hours did you work last week?', an answer of 40 hours might be reported 'to simplify communication' when the true value is known to be 42, or when the respondent is not sure how many hours were worked. Or of course, when the respondent knows precisely that the number is 40. Although ostensibly the same, rounding to convey uncertainty and rounding to convey ambiguity can have different meanings. As discussed by Manski and Molinari (2010), ambiguity arises when a forecaster feels unable to assign precise probabilities to certain events, such as future inflation or output growth taking on particular values, and consequently provides a rounded estimate of the expected future rate of inflation, or the probability that GDP will decline.[4] Alternatively, the respondent may be able to assign

[1] Binder (2017) calculates that nearly a half of the 219,181 responses to the monthly MSC surveys between January 1978 and December 2013 are multiples of 5%. The survey question is phrased as 'By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?'. Responses have to be integer valued, or 'don't know' to this survey question.

[2] Some of the key studies on rounding behaviour are on the reporting of ages of young children in Tanzania by Heitjan and Rubin (1990), as well as cigarette consumption, Wang and Heitjan (2008).

[3] Binder (2017) measures 'rounding' as the probability that the forecast has been rounded. The same is true (although the result holds a little less strongly) if a dummy to denote rounding to a multiple of 5 is used instead of the probability of rounding.

[4] They refer to Fischhoff and Bruine De Bruin (1999) who denote 'total ambiguity' as the situation when the

probabilities, that is, has a well-specified subjective probability density function, but the high variance leads the respondent to report a rounded figure. The latter is the RNRI motive. It will likely be difficult to distinguish between the ambiguity and uncertainty motives for rounding.

One consequence of finding that rounding conveys uncertainty is that the degree of rounding of point predictions can be used to proxy uncertainty, when point predictions are available but direct measures of uncertainty from density forecasts or histograms are not, as shown by Binder (2017). A focus of our paper is on the consequences for the accuracy of the reported numbers. For example, the worker who reports 40 hours to simplify communication makes an avoidable error of 2 hours.

Is it possible to discriminate between 'rounding to simplify communication' and 'rounding because the forecaster lacks the skill / knowledge to make a precise prediction'? We address this issue by considering whether the forecaster could have made a more accurate non-rounded forecast using information available at the time the forecast was made.[5] An affirmative finding would be consistent with the forecaster choosing to report a less accurate, rounded forecast: e.g., reporting an age of 40 rather than the true age of 42. We ask this question of the probability of decline forecasts, for which we find clear evidence of rounding.

Generally we find little evidence that the degree of rounding is correlated with an agents perception of the uncertainty she faces. Forecasts do not appear to be rounded to convey uncertainty. However, there is evidence of a correlation between rounding and forecast performance. This may be because less-well informed (or less able) respondents round more because they feel unable to provide a more precise forecast. Or alternatively, agents produce efficient forecasts given their information sets, and they then round these forecasts (possibly to simplify communication) to varying degrees.

We consider the U.S. Survey of Professional Forecasters (U.S. SPF) because it provides forecasts of the CPI at horizons similar to those of the MSC, permitting a comparison of the rounding practices of professionals and consumers. The U.S. SPF also provides probability forecasts for a smaller number of variables, in the form of histograms, as well as probabilities of the event that real GDP will decline in the coming quarters. Our assessment of rounding draws on all three types of forecasts. The SPF histograms provide a measure of perceived (or *ex ante*) uncertainty, EAU, which is distinct from realized or *ex post* uncertainty, EPU, following Clements (2014) (see also Knüppel and Schultefrankenfeld (2019)). EAU can be calculated from the histogram forecasts in advance of the outcome being revealed, whereas *ex post* uncertainty is the squared forecast error (or the MSFE) of the variable in question.[6] The measure of EAU

forecaster declares 'it's a fifty-fifty chance'.

[5] This relates to whether the forecasts are efficient, in the sense of Mincer and Zarnowitz (1969): see also Clements (2019) for an application to the U.S. SPF.

[6] We can only measure *ex ante* uncertainty directly for GDP growth and the GDP deflator, as we only have histograms for these two variables. This is a shortcoming which might hinder our ability to detect rounding to

allows us to determine whether perceived uncertainty is correlated with rounding behaviour. Because a forecaster's EAU and EPU are not closely correlated, it might be possible to discern whether perceived uncertainty and rounding are correlated, as distinct from forecast performance and rounding being correlated. Clements (2014, Table 5, p.214.) suggests EAU may be a poor predictor of EPU across individuals, suggesting some discrimination between the two might be possible.

Our findings can be summarized as follows. The U.S. SPF CPI and UR (unemployment rate) forecasts are rounded to a much lesser extent that the CPI forecasts of the consumers investigated by Binder (2017). The aggregate figures only suggest rounding if we consider rounding to be the reporting of a value to a multiples of 0.5 and suppose the respondent might have reported a value to two decimal places. Nevertheless, even without the last assumption, we show there are inter-forecaster differences in rounding. But there is little evidence that rounding is related to the perceived uncertainty surrounding the outcome, or that rounding negatively impacts forecast accuracy. A possibility is that these findings reflect the fact that any rounding of the CPI and UR forecasts is modest (certainly compared to the consumer forecasts). When we consider the U.S. SPF probability forecasts, namely the probabilities given to the event that quarterly real GDP growth will be negative, there is clear evidence of rounding to multiples of 5 and 10. Moreover, the evidence from inter-forecaster variation suggests rounding and accuracy are negatively related, but there is no association between rounding and perceived uncertainty. Evidence from individual-level analysis that exploits a respondent's rounding behaviour and forecast performance over time is in tune with the finding that rounding is associated with less accurate forecasts, but does not appear to be driven by perceived uncertainty. We show that forecasters could have made more accurate decline probability forecasts using real-time information, based on their output growth forecasts. However, a simulation study suggests that any rounding of the reported probability of decline forecasts has only a minor effect on the accuracy of the probabilities evaluated as forecasts of the binary event of a decline in output. Moreover, assuming rounding depends on the degree of uncertainty, for example, does not provide a better fit to the actual data, casting doubt on the importance of this putative effect. Finally, we adapt the model of low and high-uncertainty respondents of Binder (2017) to the probability of decline data, and derive some support for the proposition that uncertainty and rounding are positively related.

The plan of the remainder of the paper. Section 2 briefly reviews approaches to handling rounded data. In section 3 we consider the U.S. SPF CPI inflation forecasts to see whether professional forecasters exhibit similar rounding behaviour to consumers. We explore the relationship between the rounding of forecasts and uncertainty by considering time-series variation in the rounding behaviour of the forecasters *en masse* in section 3.1, as well as the variation

convey uncertainty.

between individual forecasters in section 3.2. Section 4 analyzes the probability of decline forecasts: section 4.1 considers the inter-forecaster patterns of behaviour, and section 4.2 uses time variation for individual respondents. Section 4.3 relates the probabilities of decline to (simultaneous) output growth forecasts, and section 4.4 estimates the model for the cross-section of respondents. Section 5 offers some concluding remarks.

2 Approaches to Handling Rounded Data

Much of the literature dealing with 'coarse' data makes use of multiple imputation (MI) - a simulation-based statistical technique - where the aim is to make 'statistically-valid' inference, in the sense of Rubin (1996). Multiple imputation is commonly used for missing data due to survey non-response, but coarse data refers to any data for which the precise values of the true data are not observed, including rounded or heaped data. Heitjan and Rubin (1990) and Drechsler and Kesl (2016) consider coarse data which is heaped (or rounded), as in the case of self-reported age data. Often the aim is to determine the size and statistical significance of a putative coarsely-observed explanatory variable. Taking the rounded data at face value may not be appropriate.

Following Heitjan and Rubin (1990), Drechsler and Kesl (2016) undertake MI by supposing there is a model for the coarse variable of interest (e.g., that the conditional distribution of the variable is normal given some covariates), and also for the degree of rounding, given that typically rounding may occur to different degrees. The degree of rounding is determined by a latent variable, which is also normally distributed conditional on some covariates. As this variable crosses various thresholds higher degrees of rounding are invoked - the model is an ordered probit over an assumed set of possible degrees of rounding.

Applying MI in the context of analyzing the rounding behaviour of survey respondents would require a specification of the model for the variable of interest (the forecast) and for the determinants of rounding behaviour. But surveys of expectations are typically undertaken without recourse to anything other than the forecasts (and actual values) typically because we do not know how the forecasts have been made (in terms of the models, techniques and judgment which have been applied). If it were the case that the respondent is unable to assign probabilities to certain events (ambiguity), then it is moot whether it is sensible to talk of 'true' forecast values. In most of the paper we do not attempt modelling, but an exception is when the forecast probabilities are related to the output growth forecasts. For the most part we consider what can be learnt from considering the relationships between rounding, and *ex ante* and *ex post* uncertainty, using the reported forecast data (the potentially rounded data), and measures derived from the histograms.

Binder (2017) is an attempt to model the forecast generation process and the rounding decision: respondents are assumed to be of two types, either high-uncertainty or low-uncertainty, and high-uncertainty forecasters choose responses rounded to multiples of 5, and the low-uncertainty forecasters choose integer-valued forecasts. That is, she makes the implicit assumption of RNRI - high-uncertainty agents choose rounder responses. In section 4.4 we apply her approach to the probabilities of decline.

When an individual makes multiple responses to the same survey, or responds to multiple surveys over time, Manski and Molinari (2010) suggest using the pattern of rounding of responses to infer an individual's rounding practice, and provide an algorithm to generate interval data to replace the reported rounded values. This obviates the need to model the data generating process and rounding behaviour, and was applied to the SPF probability of decline forecasts by Clements (2011).

A number of papers have considered whether the U.S. SPF forecasters round their probability forecasts, but from a different perspective. Both Engelberg, Manski and Williams (2009) and Clements (2011) consider whether rounding accounts for apparent inconsistencies between the histograms and point predictions (in the case of the former), or between these forecasts and the decline probabilities (in the case of the latter).

3 Comparison of Consumers and Professional Forecasters Rounding of Inflation Expectations

We use the U.S. Survey of Professional Forecasters (SPF). The SPF is a quarterly survey of macroeconomic forecasters of the U.S. economy that began in 1968, administered by the American Statistical Association (ASA) and the National Bureau of Economic Research (NBER). Since June 1990 it has been run by the Philadelphia Fed, renamed as the Survey of Professional Forecasters (SPF): see Croushore (1993). The SPF is made freely available by the Philadelphia Fed, allowing results to be readily reproduced and checked by other researchers. Its constant scrutiny is likely to minimize the impact of respondent reporting errors. An academic bibliography of the large number of published papers that use SPF data is maintained[7] and listed 101 papers as of January 2019.

The MSC consumer inflation expectations analyzed by Binder (2017) are surveyed every month, and refer to "the next 12 months". The closest inflation forecasts in the SPF are the annual fourth-quarter over fourth-quarter CPI inflation forecasts for the current year. We consider the 154 quarterly surveys from 1981:3 to 2019:4, and consider the forecasts from the 127 individuals who responded to a minimum of 12 surveys. The survey also reports these forecasts for the next year, and we analyze these as well. [8] These forecasts are fixed-event in

[7] http://www.phil.frb.org/research-and-data/real-time-center/survey-of-professional-forecasters/academic-bibliography.cfm.

[8] Also reported are the annualized quarter-over-quarter percent changes of the quarterly average price index

nature, in the sense that the current-year forecasts made in the surveys Q1 to Q4 are of the same target ("event") with a shortening horizon as the year progresses. [9] We also considered the annual unemployment rate forecasts. These are again of the current year and next year, but are the annual averages of the underlying monthly levels. [10]

Throughout the paper, actual values are taken from the Real-Time Data Set for Macroeconomists (RTDSM).[11] For example, for the current year forecasts for the surveys in 2005, we use the data available in the 2006:Q1 vintage to construct the actual 2005:Q4 on 2004:Q4 inflation rate, and for the annual average 2005 unemployment rate. For the next year forecasts from the same surveys the actuals are taken from the 2007:Q1 vintage of data.[12] However, the revisions to UR and CPI are typically small, compared to the revisions made to the national accounts data (e.g., GDP and the GDP deflator) and so the choice of actual values is likely to be inconsequential. This will not true of the National Income and Product Accounts data such as GDP, which we use subsequently, and which are subject to significant revisions - Clements and Galvão (2019) provide a recent review.

Neither set of forecasts is restricted to be integer-valued. The forecasts were recorded to one decimal place prior to 1990, and thereafter to up to two decimal places. We assume a forecasts is rounded if it is a multiple of 0.5%, and so takes on one of the following distinct values $\{\ldots, 2.0, 2.5, 3.0, 3.5, \ldots\}$. Table 1 records the total number of forecasts of each variable, and the proportion which are expressed as a multiple of 0.5, or 0.1 (i.e., given to one decimal place). If forecasts were recorded to one decimal place, then multiples of 0.5% would be expected to arise 20% of the time if no special significance were attached to such numbers. The observed proportions of multiples of 0.5 (of the forecasts given to one decimal places) are not much greater than this. Even so, we establish in section 3.2 that there are differences between respondents, which we investigate.

Moreover, because forecasts could have been recorded to two decimal places (after 1990), then we would only expect multiples of 0.5 to arise 0.2% of the time, a factor of a hundred less than we observe. However, one might be skeptical as to whether forecasts to two decimal places are meaningful.

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level. But annual forecasts, rather than quarterly forecasts, would appear to better match the MSC consumer expectations.

[9] See e.g., Nordhaus (1987) and Clements (1995) for a discussion of the evaluation of such forecasts.

[10] As of 2009:Q2, density projections for the civilian unemployment rate were also elicited.

[11] Available at:
https://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data
The RTDSM allows us to use the data vintages which were available at a specific point of time.

[12] For the CPI, the first quarterly data vintage is 1994:Q3, so the actuals up to and including 1994 are from the 1995:Q1 vintage, thereafter they are as described in the text.

3.1 Aggregate measures of rounding

We begin by considering an aggregate index of rounding calculated as the proportion of responses to each survey which are a multiple of 0.5, henceforth denoted by the shorthand 'M5'. We run regressions of the time series of the proportion of rounded responses on dummies to denote the quarter of the year of the survey. Given the fixed-event nature of the forecasts, the quarter of the year of the survey determines the forecast horizon. Because uncertainty generally increases in the forecast horizon, the literature discussed in the Introduction suggests the quarter of the survey should be a significant determinant of the degree of rounding if uncertainty and the degree of rounding are related. In addition to the effect from the shortening horizon as the survey quarter moves through the year, we include *ex ante* measures of macro uncertainty, which typically move counter-cyclically. Macro uncertainty at time t is measured by the cross-sectional median of the individual histogram variances of GDP deflator inflation, for the CPI regressions, and by the histogram variances of GDP growth, for the unemployment rate. (We use either the current-year or next-year histograms as appropriate to match the current and next-year CPI and UR forecasts).

The regression results in table 2 for current year CPI indicate none of the survey-quarter dummies, or macro uncertainty, are significant at the 5% level. (We include a constant and three dummies, for Q2, Q3 and Q4). This is true for the whole sample, as well as for the various sub-samples we consider.[13] A time trend was included and this is generally negative and significant when the dependent variable is the proportion of M5 forecasts, suggesting a long term move away from M5. This coincides with more forecasts being reported to two decimal places over the period (for both variables), and when we divide the dependent variable by the proportion of forecasts reported to one decimal place, the size of the coefficient on the trend is reduced, and is insignificant for the period before 1990:4, and after 2005:4.

For the next-year CPI forecasts (table 3), the dummy for Q4-surveys is negative and statistically significant (for the whole sample, and for a number of the sub-samples, but not the post-Crisis period), suggesting the degree of rounding is lower (than for the Q1-surveys), consistent with the view that respondents round less when the horizon is shorter. Although as noted, a similar phenomenon is not observed for the current-year CPI forecasts. For the next-year CPI forecasts we also find a significant, positive effect from macro-uncertainty for the whole period when the dependent variable is expressed as a proportion of the forecasts reported to one decimal place.

For the current-year unemployment rate forecasts the survey-quarter effect is insignificant

[13] The sub-samples are selected as follows. The survey changed hands in 1990:3, and was subsequently administered by the Phildadelphia Fed, with new operating procedures. We consider the periods before and after the change of administration separately. (Engelberg *et al.* (2009) only consider the post 1990 period as being potentially more reliable). We then split the post-1990 period roughly in half to see whether there were any effects of the Financial Crisis.

(see table 4), but macro-uncertainty is significantly negative for the period, for both definitions of the dependent variable, which is contrary to the conventional wisdom, and is explored below by exploiting inter-forecaster variation.

Table 5 shows the results for the next year forecasts. There is evidence that the degree of rounding is lower for the Q4 surveys, matching the finding for the next-year CPI forecasts. The Q4 dummy is statistically significant at the 5% level for the whole period, and at the 10% level for some of the sub-periods. But unlike for the next-year CPI forecasts, there is no evidence that macro-uncertainty affects the degree of rounding.

To conclude, the current-year forecasts of both variables do not appear to be influenced by seasonal variation in uncertainty from the fixed-event nature of the forecasts. The rounding of the next-year forecasts is lower for the less uncertain short-horizon Q4 forecasts. Somewhat surprisingly business-cycle variation in macro uncertainty is negatively associated with the rounding of current-year UR forecasts. It is positively associated with next year inflation forecasts.

The evidence based on aggregate data is mixed and does not always point in the same direction. Individuals' rounding behaviour may respond to their own perceptions of the uncertainty of the outlook, and the relationship between uncertainty and rounding may vary across individual. With this in mind, we turn to an individual-level analysis of rounding behaviour. We exploit variation across forecasters in terms of perceptions of uncertainty, and propensities to round, to explore the relationship between rounding and uncertainty for professional forecasters.

3.2 Variation across individuals

Figures 1 to 4 depict the cross-sectional variation in the propensity to round, where the propensity to round is calculated as the proportion of the respondent's forecasts which are an exact multiple of 0.5. Each bar shows the proportion of forecasts rounded by a respondent, and the respondents are ranked by least to most. A horizontal line would indicate no inter-forecaster variation in rounding propensity. For both variables and target periods, there are clear differences across forecasters - some do not round, while those who round the most do so for between 40 and 60% of their forecasts (with outliers who always round).

As discussed in the introduction, if more able respondents round less, we ought to find that forecasters who round more (as measured by the proportion of their forecasts which are M5) make less accurate forecasts on MSFE. Another hypothesis was that those who perceive greater uncertainty (EAU) would be expected to round more, and in addition, to record larger MSFEs if their perceptions of the uncertainty surrounding the outcome are accurate. We control for the economic conditions the respondents faced. Otherwise, those who were active during difficult times, such as the 2008-9 period, would have larger uncertainty measures for this very reason. Both our measures of EAU and EPU are calculated as relative measures. For the *ex post* uncertainty, we normalize forecast errors by dividing by the average degree of difficulty experienced in forecasting at that point in time, as measured by the (square root) of the cross-sectional MSFE. This follows D'Agostino, McQuinn and Whelan (2012) and Clements (2014). Specifically, if $e_{i,t}$ denotes the forecast error made by individual i, in response to forecast survey t (the horizon is left unspecified for simplicity), we calculate the normalized forecast errors as:

$$\widetilde{e}_{i,t} = \frac{e_{i,t}}{\sqrt{\frac{1}{N_t}\sum_{j=1}^{N_t} e_{j,t}^2}} \tag{1}$$

where N_t is the number of respondents to survey t. This approach implements an *ex post* adjustment to each forecast error, based on the realized forecast loss, to prevent inter-forecaster comparisons of accuracy being distorted. For each of the 127 respondents, we then calculate MSFE_i as the average squared error of $\widetilde{e}_{i,t}$ over all the surveys t to which individual i responded. For the EAU of respondent i, we first divide the estimate of the histogram variance[14] at time t, denoted $\hat{\sigma}_{i,t}^2$, by the cross-sectional average of all the active participants at t:

$$\widetilde{\sigma}_{i,t}^2 = \frac{\hat{\sigma}_{i,t}^2}{\frac{1}{N_t}\sum_{j=1}^{N_t} \hat{\sigma}_{j,t}^2}$$

and then take the average of $\widetilde{\sigma}_{i,t}^2$ over all the surveys to which i contributed. We have estimates of EAU for output growth and GDP deflator inflation. For both variables the histograms are of the annual growth rates in the current calendar year relative to the previous year, and of the next year relative to the current year. We use the output growth variance estimates to measure EAU regarding the unemployment rate, and the GDP deflator for CPI inflation.[15]

We test for significant correlation between the propensity to round, EAU, and forecast accuracy, allowing that the relationship need not be linear. We consider the relationship between the ranks - whether the individuals who are highly ranked in terms of rounding, are also highly ranked in terms of forecast accuracy, or in terms of their perceived uncertainty. The Spearman rank correlation r lies between -1 and 1, where 0 indicates no relationship. The rank correlation

[14] We estimate the variances by fitting normal distributions to the histograms when non-zero probability mass is assigned to 3 or more (consecutive) intervals, and triangular distributions otherwise (as described by Engelberg *et al.* (2009, p.37-8)). See Clements (2019) for a discussion.

[15] Because the CPI inflation forecasts are of Q4 on Q4, the EAU and CPI forecast target periods are not exactly aligned.

given by:

$$r = 1 - \frac{6R}{N\left(N^2 - 1\right)}$$

where R is the sum of squared differences between the ranks (of the forecasters by degree of rounding, and by the size of MSFE, say).[16]

Table 6 shows some evidence that rounding and forecast performance are negatively associated (i.e., the degree of rounding and MSFE are positively related) for the longer horizon forecasts (the 'next year' forecasts), but not for short horizons. We formally reject the null of no correlation for the UR forecasts (at the 10% level in a two-sided test), but would only reject at the 20% level for the CPI next-year forecasts.[17]

The results based on the cross-sectional variation also suggest that rounding is not related to the perceived uncertainty surrounding the outcome. Contrary to the finding in Binder (2017) for consumers, there is no evidence that professional forecasters who round more tend to be individuals with higher measures of *ex ante* forecast uncertainty. As anticipated in the Introduction, there is very little evidence that perceived and realized uncertainty are correlated across respondents: see the last panel of table 6. For current-quarter inflation there is some evidence of a positive correlation, but we do not reject the null at conventional significance level (such as the 10% level in a two-sided test).[18]

In summary, neither the aggregate time-series regressions or the rank correlations between individuals' estimates of rounding propensity and perceptions of forecast accuracy lend unequivocal support to the proposition that professional forecasters round more when they face a more uncertain outlook. There is no evidence that rounding of forecasts of the current year (either CPI or UR) is positively associated with less accurate forecasts.

So far we have considered point predictions of CPI and UR. As noted, the evidence of rounding of these forecasts is much less persuasive than the evidence for consumers. In the next

[16] It is common to calculate the Fisher transformation,

$$F(r) = \frac{1}{2}\ln\frac{1+r}{1-r}$$

such that $z = F(r) . \sqrt{\frac{N-3}{1.06}} \sim N(0,1)$ under the null of statistical independence. As well as reporting r, we report the probability of observing a test statistic less than that obtained under the null hypothesis (of a zero correlation). Probabilities less than 0.05 or greater than 0.95 indicate rejections of the null in a two-sided test at the 10% level. A probability less than 0.05 suggests a negative correlation, and one greater than 0.95 a positive correlation.

[17] We group the current-year forecasts together, and similarly for the next year forecasts. The former have aproximate horizons of 1 to 4-quarters ahead, and the latter of 5 to 8 quarters ahead. In principle one could consider the relationship between rounding and accuracy at a particular horizon, e.g., 8 quarters ahead, if we considered only the next-year forecasts made in response to Q1 surveys. In practice this would mean that the estimates of rounding proportions and MSFE-accuracy would be based on only as quarter as many forecasts.

[18] A caveat is that neither the variable definitions (e.g., CPI versus GDP deflator, and UR versus GDP), target periods or horizons line up perfectly, but the evidence is consistent with Clements (2014) where there are no such differences.

section we consider the probability forecasts. We know from the literature that the forecast probabilities of decline in output exhibit clear evidence of rounding.

4 Probability forecasts

A number of studies consider the SPF and rounding. Clements (2011) considers whether rounding accounts for the mismatch documented by Clements (2009) between respondents' probability forecasts of a decline in real output, and their histograms for annual real output growth. The mismatch arises when the reported probability forecasts are taken at face value. Clements (2011) shows that the mismatch remains if we allow for plausible patterns of rounding behaviour. Clements (2014) shows that rounding of the probability distributions can not account for the tendency of forecasters' perceptions of uncertainty (*ex ante* uncertainty, calculable from the histograms) to exceed realized or *ex post* uncertainty at within year horizons. Undoing the rounding, as in Engelberg *et al.* (2009, Appendix, pp. 40-1), for example, would further inflate forecaster perceptions relative to the actual.

Clements (2011) considered the U.S. SPF quarterly surveys from 1981:3 to 2010:1, and the 2976 sets of probability of decline forecasts from the 100 regular forecasters respondents, defined as those who responded to 12 or more surveys.[19] The probability of decline forecast $p_{j,t}^{h}$ is the forecast probability reported by respondent j, to the survey in quarter t, of the event that the level of real output will be lower in quarter $t+h$ than $t+h-1$. The respondents provide forecasts for $h=0,1,\ldots 4$, where $h=0$ refers to a forecast of a decline in output in the current quarter (the survey date quarter) relative to the previous quarter, and $h=4$ is a forecast of the same quarter a year ahead relative to three quarters ahead.

Table 7 reports similar degrees of rounding when the forecast data set is updated to include some 60% more forecasts. 85% of the current quarter forecasts are reported as multiples of either 5 or 10. This edges up to 88% for the forecasts of a quarterly decline a year ahead. This suggests little evidence that rounding is done to convey uncertainty: the proportion of longer-horizon forecasts which are rounded is similar to that for the current year forecasts. In addition, the proportion of forecasts which are multiples of 10 is 52 to 55% for all horizons.

However, the higher proportion of zero-probability current-quarter forecasts might partly camouflage evidence of rounding to convey uncertainty. Excluding the zero-forecasts leads to an increase in rounding to a multiple of 5 from 64% to 83% as the horizon increases.[20]

Figure 5 ranks each of the 127 forecasters we consider from the individuals who round the least to the two who round all their forecasts to a multiple of 10. The vast majority round

[19] Regular respondents are less likely to make errors in filling in the survey questionnaires compared to occasional respondents.

[20] One might expect the forecast probability of a decline to approach the relative frequency of declines as the horizon increases, resulting in fewer zero-probability forecast.

between 20% and 80% of their forecasts. We consider whether the inter-forecaster differences in propensities to round are associated with differences in average perceptions of uncertainty. That is, whether some individuals beliefs are persistently more uncertain than those of others. Also of interest is whether the differences in rounding behaviour cause some respondents to be worse than others. The differences across individuals in rounding behaviour suggests that an analysis of inter-forecaster variation could be informative about the relationship between rounding and uncertainty.

4.1 Inter-forecaster Variation

To assess forecast accuracy, the probability forecasts are compared to the event of a decline in real GDP calculated from the data vintage available at the time. For example, the current quarter forecasts from the 1981:3 survey are compared to the event of a decline between GDP in 1981:3 and 1981:2, both taken from the 1981:4 data vintage. And the 1981:3 survey $h = 4$ forecasts are compared to the change between 1982:2 and 1982:3 actual values from the 1982:4 vintage of data. We score the probability forecasts using the Brier or quadratic probability score (QPS: Brier (1950)), which is simply the expected squared error $E\left[(p-y)^2\right]$, where p is the probability, and y takes the value of 1 when the event occurs, and zero otherwise.[21] For a sequence of probability forecasts and outcomes, $\{p_t, y_t\}$, $t = 1, \ldots, n$, these scores are calculated as:

$$\text{QPS} = \frac{1}{n}\sum_{t=1}^{n}(p_t - y_t)^2 . \tag{2}$$

As for the MSFE for the point forecasts, we normalize the QPS for individual i by:

$$\text{QPS}_i = \frac{1}{n_i}\sum_{t \in N_i}\left(\frac{p_{ti} - y_t}{\sqrt{\frac{1}{N_t}\sum_{j=1}^{N_t}(p_{t,j} - y_t)^2}}\right)^2$$

where N_i is the set of surveys (numbering n_i) that i responded to, and N_t is the dimension of the cross-section for survey t.

If a forecast probability of zero is assumed to reflect a rounded forecast, then Table 8 suggests rounding and accuracy are negatively related (i.e., the correlation between rounding propensity and forecast loss is positive) for the longer-horizon $h = 4$ forecasts, but not for the current quarter forecasts. In addition, more perceived uncertainty is associated with less rounding. Treating the zero probability forecasts as being rounded is problematic if they represent underlying beliefs that the event (a decline in output) is extremely unlikely. If we exclude zero probabilities from the definition of rounding, the variation across individuals unambiguously shows rounding

[21] We use QPS rather than the logarithmic probability score (LPS: see Brier (1950) and Good (1952)), defined as $E\left[-y\log(f) - (1-y)\log(1-f)\right]$ because of the occurrence of zero-probability forecasts.

is associated with worse forecasts (rounding and QPS are positively related), and removes the anomalous negative relationship between (histogram) uncertainty and rounding. Histogram uncertainty and rounding are not correlated across individuals.

4.2 Individual Regressions

For the respondents who made a reasonable number of returns we estimate individual regressions. The evidence in section 4.1 based on inter-forecaster variation suggested rounding is not related to uncertainty, but does worsen forecast accuracy. This evidence does not require a linear relationship between uncertainty and rounding. We simply consider whether respondents who round more also have higher (or lower) perceptions of uncertainty than average. Or tend to produce forecasts of higher (or lower) than average accuracy. But we have not exploited the variation in an individual's behaviour over time. For respondents with many forecasts over the period, this variation may be informative. In this section we consider the time variation via individual regressions.

4.2.1 Rounding and Perceived Uncertainty

For each respondent who made at least 40 probability of decline forecasts, we estimated a logit regression for the dummy variable of rounding to 10, with the estimated histogram standard deviation as an explanatory variable. We considered the current-quarter probabilities of decline, and defined uncertainty as the histogram standard deviation of the current-year histogram forecasts, and the four-quarter probability of decline forecasts, using the next-year output growth histograms. We do not report the results, because the uncertainty variable was only statistically significant in a handful of cases, consistent with type 1 error. These findings are consistent with the results based on inter-forecaster variation in table 8, when zero is not treated as a rounded forecast. The individual regression results did not depend on the treatment of the zero probability forecasts.

4.2.2 Rounding and Forecast Accuracy

We regressed the normalized QPS score on a dummy for rounding. See table 9 for the results for the current-quarter and +4 quarter ahead probability of decline forecasts, when we do not consider zero forecasts as rounded forecasts. Table 10 shows results for the same horizons when the rounding dummy takes the value of 1 for a probability of decline forecast of zero: zero forecasts are considered to have been rounded. If we do not consider zero forecasts as rounded forecasts, the rounding dummy is statistically significant for over 60% of the individual respondents (22 of the 36, at the 5% level), for the current-quarter forecasts, and in every such instance is positive, signifying that rounding is associated with less accuracy. If we include zeros as rounded values, the relationship between accuracy and rounding is weakened - the number of regressions in which the dummy variable is statistically significant more than halves. This is consistent with the forecasts of zero reflecting beliefs that the event is extremely unlikely (as opposed to rounding), and this turning out to be accurate. Hence the evidence based on the individual regressions in tables 9 and 10 for the current-quarter forecasts is in line with table 8. Forecasts of zero typically reflect correct beliefs that a decline is unlikely, and excluding zero forecasts from the set of rounded forecasts strengthens the finding that rounding has a significant, deleterious effect on forecast accuracy for the majority of the respondents.

For the +4 quarter ahead forecasts the relationship between rounding and accuracy is weaker, and only statistically significant for around a quarter of the respondents. Rounding is less costly, as might be expected, given that respondents 'true' forecasts of decline four-quarters ahead are likely much less precise than the current-quarter forecasts (i.e., greater ambiguity).

We stress that the dependent variable at time t is the QPS value for individual i at time t divided by the cross-sectional mean of QPS at time t. Hence the coefficient of the rounding dummy records the increase/decrease in i's relative score from rounding.

4.3 Modelling Probability of Decline Forecasts Using Output Growth Forecasts

Some evidence can be brought to bear on whether forecasters round to simplify communication, by considering together the probability of decline and output growth forecasts typically reported by each forecaster. This allows us to approximate an unrounded probability of decline forecast whenever a pair of these forecasts is reported. We suppose that each individual's quarterly growth rate forecast w_{ith} is the mean of a gaussian density forecast, $N\left(w_{ith}, \sigma^2_{w,ith}\right)$ (see, for example, Clements (2008)) then:

$$p_{ith} = \Pr\left(W_{t+h} < 0\right) = \Phi\left(\frac{-w_{ith}}{\sigma_{w,ith}}\right). \tag{3}$$

For the 4th-quarter of the year surveys the histogram forecasts can be used to estimate $\sigma_{w,ith}$, as explained in Clements (2009). For the other quarters of the year this is not possible because of the fixed-event nature of the U.S. SPF histograms. There are ways of calculating approximate fixed-horizon density forecasts (and thus variance estimates), as suggested by Ganics, Rossi and Sekhposyan (2020). A simpler approach may be to estimate the relationship between p and w non-parametrically, using the Nadaraya-Watson regression to estimate the conditional expectation of p given w, $g\left(w\right) = E\left(p_{ith} | w_{ith} = w\right)$. For individual j at survey time t =1991:3, the non-parametric estimator of $g(w)$ is:

$$\hat{p}_{j,t=1991:3,h} = \hat{g}\left(w_{j,1991:3,h}\right) = \frac{\sum_i \sum_{t=1981:3 \text{ to } 1991:2} k\left(\frac{w_{ith}-w_{j,1991:3,h}}{b}\right) p_{ith}}{\sum_i \sum_{t=1981:3 \text{ to } 1991:2} k\left(\frac{w_{ith}-w_{j,1991:3,h}}{b}\right)} \tag{4}$$

where $k()$ is the kernel function and b the bandwidth. We then extend the model estimation period so that the summation includes 1991:3, and estimate the conditional expectations at $w_{j,1991:4,h}$, for each j, [22] and so on.

When the reported forecasts p are rounded, they measure the 'true' forecasts, say, p^0, with error, $p_{ith} = p^0_{ith} + r_{ith}$, say. Equation (4) can be viewed as a model of the relationship between the true unrounded forecasts p_0 and w if we substitute $p_{ith} = p^0_{ith} + r_{ith}$ into (4) and assume that the term:

$$\frac{\sum_i \sum_{t=1981:3 \text{ to } 1991:2} k\left(\frac{w_{ith}-w_{j,1991:3,h}}{b}\right) r_{ith}}{\sum_i \sum_{t=1981:3 \text{ to } 1991:2} k\left(\frac{w_{ith}-w_{j,1991:3,h}}{b}\right)}$$

is negligible and can be ignored. Whereas ignoring rounding (taking the reported value at face value) may give misleading results when the object of the analysis is to learn about the size and significance of an explanatory variable in a structural model, our aim is more modest, to obtain a simple forecasting model for p^0.

The last two columns of table 9 show the ratio of the QPS for $\hat{p}$ to that for p for each respondent. (This statistic does not depend on whether a zero forecast is assumed to be a rounded forecast or not.) We report results for the subset of forecasts t between 1991:3 and 2019:4 for which both $\hat{p}_{it}$ and p_{it} exist for respondent i.[23] $\hat{p}$ is more accurate on QPS than p more often than not: the current-quarter predicted probabilities are more accurate for three-quarters of the forecasters, and over 10% more accurate for nearly half of the respondents.[24]

These findings are consistent with the proposition that probability of decline forecasts are rounded to simplify communication. This is because more accurate non-rounded forecasts were readily available, based only on the individual's quarterly output growth forecasts, and a simple model linking the two (which can be estimated from the history of w and p forecasts through $t-1$). That is, using only information available to the forecaster at the time the forecast was made, superior (non-rounded) forecasts could have been made.

[22] Hence the approach is real time in the sense that at each t, the forecast $\hat{p}_{ith}$ uses forecasts of w and p through $t-1$, which will be known to each respondent, and the respondent i's time t forecast w_{ith}. If $\hat{p}_{ith}$ is superior to p_{ith} in terms of forecast accuracy, then the reported forecasts are inefficient in the sense that they do not exploit all readily available information: see Mincer and Zarnowitz (1969).

[23] The first survey is 1991:3 because the 1981:3 to 1991:2 period is used to estimate the relationship beween the forecast of quarterly output growth w_{it} and the forecast probability of decline p_{it}. Note that p_{ith} will typically be missing for some periods, and $\hat{p}_{ith}$ will be missing whenever w_{ith} is missing.

[24] We consider QPS as perhaps the simplest way of scoring the forecast probabilities: Lahiri and Wang (2013) consider other approaches.

However it may be wrong to attribute the difference in accuracy between $\hat{p}$ and p solely to the effect of rounding. It may be that respondents who are less efficient in their use of relevant information also tend to round more. Suppose that $\hat{p} = g(W = w)$ exploits useful information neglected by the respondent in producing p, such that $\hat{p}$ would be more accurate than the reported p even if the latter were not rounded. To address this issue, we use a simulation to estimate the effects of rounding on the accuracy of the forecast probabilities of decline. In the simulation, we can compare the true probabilities (denoted by p^0) to the rounded probabilities, p, assuming a particular rounding scheme, in terms of QPS: i.e. we can compare $E\left(p^0 - 1_{w<0}\right)^2$ and $E\left(p - 1_{w<0}\right)^2$. The difference between the simulation estimates of these two losses estimates the effect of rounding. Because we only observe the reported forecast probabilities, the empirical counterpart of this loss is not available. In the simulation we also estimate the difference between $E\left(p - 1_{w<0}\right)^2$ and $E\left(\hat{p} - 1_{w<0}\right)^2$. Empirical estimates of the ratio of these two losses can be calculated and tabulated as in table 9, using the non-parametric estimates of $\hat{p}$.

Details of the model are provided in the Appendix. Forecaster behaviour is given by the noisy information model,[25] and is loosely calibrated on SPF forecasts of quarterly real GDP. Probability of decline forecasts are obtained assuming gaussianity and assuming each respondent knows the variance of future output growth. We allow the forecast error variance to differ across respondents by assuming different signal precisions.[26] The model assumes forecasters are rational given the informational rigidities they face. Our simulation findings were qualitatively unaffected if instead of assuming noisy information, we allow that past values of real output growth are observed, or if we suppose that respondents are no longer rational but are subject to a behavioural bias, such as in the diagnostic expectations of Bordalo *et al.* (2018).

We investigate the consequences of three rounding behaviours. In the first, R_1, agents round to a given multiple with a prescribed probability. We assume rounding to a multiple of 0.1 with a probability of 0.4. The second R_2 supposes the probability of rounding depends on the agent's true probability, p^0, the simplest case of which is to assume the probability of rounding equals p^0. Recessionary times are typically associated with higher uncertainty, so that when the probability of a decline is higher the agent is more likely to round her forecast to reflect the higher uncertainty. Thirdly, R_3, when we assume heterogeneous agents, the probability of rounding is higher for agents with higher forecast-error variances (that is, with less precise signals). Specifically, we assume the probability of rounding is 0.4 times the ratio

[25] This noisy information model has become one of the leading models of forecast behaviour, see e.g., see Woodford (2002), Sims (2003) and Coibion and Gorodnichenko (2012, 2015), *inter alia*.

[26] See, e.g., D'Agostino *et al.* (2012) and Clements (2019, 2020) for evidence on whether some forecasters really are better than others. In terms of the noisy information model, better forecasters are the recipients of more precise signals. Bordalo, Gennaioli, Ma and Shleifer (2018) suggest that instead of interpreting the noisy signal an agent receives as reflecting 'inattentiveness', as is often done in this literature, it might be more reasonable to suppose it reflects the use of "different models or pieces of evidence".

of the agent's forecast-variance to the median error-variance across agents. Hence the median forecaster acts as under our first assumption, whereas better (worse) forecasters are less (more) likely to round.[27] Hence R_3 captures the idea that agents round to convey uncertainty, and in our model, agents correctly perceive the uncertainty they face.

The findings of the simulations are recorded in table 11. The column headed $E\left(p^0 - 1_{w<0}\right)^2$ reports the estimate of the QPS value for the true forecast probabilities, and the next three columns are the ratios of the QPS values for each of the three rounded forecasts (R_1, R_2 and R_3), to that using p^0. [28] We report the cross-sectional mean across the respondents as well as the minimum and maximum. When agents are homogeneous the three measures are close to each other, and only differ because of simulation error. The effects of rounding on event-forecast accuracy are barely perceptible, and any dependence on the precision of the signal is again barely perceptible. Because the empirical estimates are often of an order of magnitude larger than the estimates in the simulation, we conclude that the difference in accuracy between $\hat{p}$ and p for the most part does not reflect rounding, but forecast inefficiency.

Whereas rounding has only a small effect on forecasting the event that output will be lower, the simulation shows that it naturally has a much larger effect when we compare the accuracy of $\hat{p}$ and p as forecasts of p^0. The last three columns show the expected squared error of the corrected probability $\hat{p}$ (for a given rounding scheme, R_j) as a forecast of p^0 as a ratio of the expected squared error of p (for a given rounding scheme, R_j) as a forecast of p^0. The corrected forecasts are obtained from (4) by replacing p_{ith} with the forecasts obtained by applying one of the three rounding schemes. The last three columns show that on average across respondents the corrected forecasts can be much more accurate than that of the rounded forecasts for the true probabilities when the respondents are homogeneous (rows with 'Hetero. = 0'). Under heterogeneity, and when the private signals are relatively more important ($\sigma_\varepsilon = 1$, as opposed to 3), the performance of the non-parametric estimate of the respondent's unrounded probability p^0 depends on the precision of the private-information signal. For example, under noisy information the range for 'random' rounding R_1 is 0.150 to 4.698. The maximum value of 4.698 is found for the forecaster who receives the most precise signal. This respondent's rounded forecasts are more accurate than the non-parametric estimates, because of the importance of the time-t signal, which is foregone when (4) is used. As the signal precision deteriorates, the omission of the signal in estimating the non-rounded value matters less, and the ratio falls. The same is observed for the other rounding schemes. The reduction is not monotonic as the signal precision decreases, and the ratios display 'U' shaped patterns.

[27] In our simulations the forecast-error variances differ across agents when we allow heterogeneity, but do not depend on time.

[28] We report the averages of the 100 respondents, where for each respondent we calculate QPS over 10,000 replications. When the forecasters are homogeneous, this is equivalent to 1,000,000 replications of one respondent, but the distinction is meaningful when we permit heterogeneity.

To recap: we cannot directly determine the effect of rounding on event-forecasting accuracy because we only observe the reported value, which has been rounded to an unknown extent. However, out simulation shows that empirically-plausible rounding behaviours only worsen accuracy by less than half a percentage point. Hence the improvements in empirical forecast accuracy of the order of 10% from the non-parametric estimates $\hat{p}$ likely reflect the failure of the forecasters to fully utilize all available information, rather than the effects of rounding *per se*.

4.4 Model of the Cross-sectional Distribution of Probability of Decline Responses

Binder (2017) supposes reported survey responses R_{it} are generated by a combination of lower-uncertainty l-agents, who round less, and higher-uncertainty h-agents, who round more. (Note h now indicates 'high', not the forecast horizon). She uses the proportion of the two types at each time t as the basis of a time-series uncertainty index. We adapt her analysis of CPI inflation rates for the probabilities of decline.

We suppose type-h respondents choose from a set of rounded responses, S_h, and type l from a set S_l, where $S_h \subset S_l$, indicating S_h is coarser than S_l. The distribution of survey responses at t is a mix of the probability mass function (pmf) ϕ_t^l for type l-agents, with support on S_l, and ϕ_t^h, with support on S_h. For the probability forecasts, $S_l = \{\ldots, 0, 1, 2, 3, \ldots\}$ and $S_h = \{0, 5, 10, 15, \ldots\}$, where most but not all reported forecasts are integer-valued. If R_{it} is not an element of S_h, then i is not type h, but if $R_{it} \in S_h$, i could be either type. We assume the pdf of i is either $p_l(x)$ if $i \in l$ or $p_h(x)$ if $i \in h$, where $p_l(x) \sim LG(\mu_l,\sigma_l)$, and $p_h(x) \sim LG(\mu_h,\sigma_h)$, where we expect to find $\sigma_h > \sigma_l$ if uncertainty explains rounding. That is, the agents who round are those who perceive more uncertainty. LG denotes the logistic pdf density function, and replaces the normal density used by Binder. Hence:

$$\begin{aligned}
\phi_t^l &= P(R_{it} = j | i \in l) = \int_{f_{\min}^l(j)}^{f_{\max}^l(j)} p_l(x)\,dx,\ j \in S_l \\
\phi_t^h &= P(R_{it} = j | i \in h) = \int_{f_{\min}^h(j)}^{f_{\max}^h(j)} p_h(x)\,dx,\ j \in S_h
\end{aligned}$$

where $f_{\min}^l(j)$ and $f_{\max}^l(j)$ are the min and max values of the underlying forecast distribution that are rounded to the reported value j if the forecaster is type l, and similarly for $f_{\min}^h(j)$ and $f_{\max}^h(j)$ for type h. If we consider rounding to '5', for example, then for $j = 45$, say, $\left[f_{\min}^h(j=45), f_{\max}^h(j=45)\right] = [42.5, 47.5]$, and $\left[f_{\min}^l(j=45), f_{\max}^l(j=45)\right] = [44.5, 45.5]$. For j which is not a multiple of 5, say, $j = 46$, $\left[f_{\min}^l(j=46), f_{\max}^l(j=46)\right] = [45.5, 46.5]$, but $\phi_t^h(j=46) = 0$.

In period t the survey responses come from $\phi_t = \lambda_t \phi_t^h + (1 - \lambda_t)\,\phi_t^l$, and maximizing the log-likelihood $\sum_{j \in S_l} N_{tj} \log \phi_t\,(j)$ provides estimates of $\left\{\lambda_t,\, \mu_{t,h},\, \sigma_{t,h},\, \mu_{t,l},\, \sigma_{t,l}\right\}$, where N_{tj} is the number of responses $R_{it} = j$, $j \in S_l$, at time t. The number of SPF responses at each t are far fewer than those available to Binder, and so we aggregate the responses over t, and suppose $\phi_t^h = \phi^h$, $\phi_t^l = \phi^l$ and $\lambda_t = \lambda$, for all t, as well as $N_j = \sum_t N_{tj}$. This means it is not possible to consider time-series variation - for example, how the proportion of rounders evolves over time. But within this approach we can ask whether rounders perceive more uncertainty than the non-rounders, and formally test (via a standard likelihood ratio test) the null hypothesis that $\sigma_h = \sigma_l$. Figure 6 plots the estimated densities values $\hat{\phi}_t^l$ and $\hat{\phi}_t^h$, and records the estimates of the location and scale parameters of the logistic distributions. We find that $\hat{\sigma}_l < \hat{\sigma}_h$, and reject the null of equality of the σ's at any significance level. We find $\hat{\lambda} = 0.59$, indicating that over a half of the respondents are of the high-uncertainty types who tend to round their forecasts.

While our findings are consistent with those of Binder for consumers, because there are too few observations to estimate λ and the parameters of the two distributions for each t, w are unable to correlate a series of λ estimates with proxies of forecast uncertainty, and are not able to show whether or not rounding depends on uncertainty.

5 Conclusions

There is some evidence that U.S. Professional Forecasters round their point forecasts of CPI inflation and the unemployment rate, but perhaps not surprisingly to a lesser extent than Binder (2017) found for consumers' inflation forecasts. We found little evidence that forecasts were rounded to a greater extent in response to higher perceived uncertainty about future outcomes, at odds with the findings for consumer inflation forecasts. However, there is some evidence that respondents who are more prone to round their forecasts produce less accurate forecasts.

By way of contrast, the event-probability forecasts (the forecast probabilities of a decline in output) are clearly rounded. Around 85% of the probabilities that output will decline in the current quarter are reported as multiples of 5, and over a half are multiples of 10. If we consider inter-forecaster variation, or individual regressions of accuracy on rounding, we find that rounding is correlated with worse event-forecast accuracy. However, our findings might reflect the fact that worse (less accurate) forecasters round more, rather than the degree of rounding *per se* worsening accuracy.

A respondent's probability of decline and output growth forecasts of the same quarter ought to be closely related, and we exploit this to generate series of non-rounded probability forecasts for all the respondents. These are found to be up to 10% more accurate on QPS than the reported probabilities for over a half of respondents. If respondents' behaviour can be approximated by our model (and they make use of the information on output growth forecasts), then subsequent rounding of these 'efficient' forecasts accounts for the reduction in accuracy.

But if respondents' forecasts are not efficient, it may still be the case that less good forecasters round more.

We overcome this impasse by simulating the loss from rounding for a set of efficient forecasters, under a number of assumed rounding schemes. The size of the simulated losses from rounding are not commensurate with the empirical estimates. Rounding of itself has a relatively minor impact on *event*-forecast accuracy, and the assumption that respondents round otherwise efficient forecasts is untenable.

We conclude that rounding of probability forecasts appears to matter little for event-forecasting. It would matter if the rounded forecasts were compared to the true probabilities, but the latter are of course only available in simulation studies. We have not considered probabilities in the form of survey histogram forecasts, and leave this issue for future research.

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Table 1: Rounding of CPI and UR annual point forecasts

	no. forecasts. (#)	M5/#	M1/#
CPI current year (Q4 on Q4)	4709	0.170	0.733
CPI next year (Q4 on Q4)	4432	0.193	0.783
UR current year	4885	0.175	0.765
UR next year	4757	0.176	0.785

Table 2: Aggregate time-series rounding results for CPI current year (Q4 on Q4)

	Constant	Q2	Q3	Q4	Time Tr.	Macro Unc.	R^2
Dep. var.: Proportion of forecasts rounded to M5 at each t							
1981:3 to 2019:4	0.348	0.000	0.013	0.005	-0.002	-0.064	0.409
	0.000	0.999	0.512	0.833	0.000	0.225	.
1981:3 to 1990:3	0.315	0.067	0.060	-0.033	-0.002	-0.049	0.117
	0.012	0.239	0.381	0.691	0.204	0.680	.
1990:4 to 2019:4	0.398	-0.021	-0.004	0.011	-0.002	-0.112	0.460
	0.000	0.213	0.844	0.648	0.000	0.096	.
1990:4 to 2005:4	0.451	-0.027	-0.018	0.019	-0.003	-0.141	0.244
	0.000	0.366	0.565	0.669	0.001	0.085	.
2006:1 to 2019:4	0.303	-0.014	0.017	0.012	-0.002	0.014	0.333
	0.003	0.447	0.295	0.535	0.005	0.915	.
Dep. var.: Proportion relative to those reported to 1 decimal place							
1981:3 to 2019:4	0.269	0.006	0.029	0.038	-0.001	0.002	0.108
	0.000	0.782	0.243	0.175	0.008	0.971	.
1981:3 to 1990:3	0.315	0.073	0.066	-0.037	-0.002	-0.060	0.126
	0.012	0.199	0.344	0.657	0.306	0.603	.
1990:4 to 2019:4	0.289	-0.016	0.008	0.047	-0.001	-0.034	0.121
	0.000	0.462	0.753	0.125	0.021	0.652	.
1990:4 to 2005:4	0.426	-0.027	-0.021	0.023	-0.002	-0.123	0.185
	0.000	0.405	0.528	0.622	0.008	0.137	.
2006:1 to 2019:4	0.191	-0.007	0.042	0.083	0.000	0.170	0.138
	0.290	0.834	0.254	0.052	0.728	0.423	.

For each sample period, the first row gives the parameter estimates, and the second row the p-value of the null that the coefficient equals zero, using heteroscedasticity-consistent standard errors. The measure of uncertainty is the cross-sectional median of the variance estimates from the individuals' current-year annual GDP deflator growth histograms.

Table 3: Aggregate time-series rounding results for CPI next year (Q4 on Q4)

	Constant	Q2	Q3	Q4	Time Tr.	Macro Unc.	R^2
Dep. var.: Proportion of forecasts rounded to M5 at each t							
1981:3 to 2019:4	0.300	-0.027	-0.019	-0.055	-0.001	0.044	0.336
.	0.000	0.221	0.410	0.013	0.000	0.246	.
1981:3 to 1990:3	0.129	-0.024	-0.008	-0.028	0.001	0.150	0.134
.	0.265	0.689	0.899	0.646	0.553	0.049	.
1990:4 to 2019:4	0.365	-0.033	-0.027	-0.063	-0.001	-0.030	0.294
.	0.000	0.157	0.293	0.008	0.000	0.633	.
1990:4 to 2005:4	0.367	-0.061	-0.054	-0.080	-0.001	-0.041	0.124
.	0.000	0.100	0.210	0.037	0.126	0.605	.
2006:1 to 2019:4	0.215	-0.002	0.003	-0.042	-0.001	0.067	0.167
.	0.082	0.933	0.904	0.146	0.352	0.511	.
Dep. var.: Proportion relative to those reported to 1 decimal place							
1981:3 to 2019:4	0.203	-0.019	-0.015	-0.036	0.000	0.103	0.091
.	0.000	0.463	0.568	0.194	0.672	0.014	.
1981:3 to 1990:3	0.087	-0.008	-0.005	-0.024	0.002	0.170	0.142
.	0.526	0.901	0.931	0.702	0.398	0.049	.
1990:4 to 2019:4	0.258	-0.028	-0.023	-0.042	0.000	0.016	0.024
.	0.000	0.335	0.436	0.177	0.939	0.814	.
1990:4 to 2005:4	0.337	-0.061	-0.050	-0.077	-0.001	-0.031	0.089
.	0.001	0.119	0.263	0.057	0.468	0.699	.
2006:1 to 2019:4	-0.162	0.008	0.013	0.009	0.002	0.332	0.048
	0.510	0.849	0.732	0.860	0.142	0.084	.

For each sample period, the first row gives the parameter estimates, and the second row the p-value of the null that the coefficient equals zero, using heteroscedasticity-consistent standard errors. The measure of uncertainty is the cross-sectional median of the variance estimates from the individuals' next-year annual GDP deflator growth histograms.

Table 4: Aggregate time-series rounding results for UR current year

	Constant	Q2	Q3	Q4	Time Tr.	Macro Unc.	R^2
Dep. var.: Proportion of forecasts rounded to M5 at each t							
1981:3 to 2019:4	0.420	-0.021	-0.038	-0.043	-0.002	-0.203	0.112
.	0.000	0.384	0.287	0.456	0.000	0.005	.
1981:3 to 1990:3	0.063	0.014	0.069	0.209	0.001	0.098	0.093
.	0.768	0.815	0.512	0.293	0.739	0.598	.
1990:4 to 2019:4	0.476	-0.014	-0.026	-0.063	-0.002	-0.220	0.155
.	0.000	0.652	0.583	0.359	0.000	0.047	.
1990:4 to 2005:4	0.376	0.003	-0.017	-0.030	0.000	-0.286	0.046
.	0.017	0.943	0.805	0.776	0.853	0.039	.
2006:1 to 2019:4	0.108	-0.001	0.022	-0.012	0.000	0.098	0.052
.	0.276	0.978	0.713	0.884	0.555	0.526	.
Dep. var.: Proportion relative to those reported to 1 decimal place							
1981:3 to 2019:4	0.377	-0.017	-0.032	-0.039	-0.001	-0.172	0.030
.	0.000	0.582	0.456	0.567	0.042	0.031	.
1981:3 to 1990:3	0.062	0.014	0.074	0.209	0.001	0.098	0.092
.	0.771	0.816	0.488	0.293	0.727	0.600	.
1990:4 to 2019:4	0.424	-0.011	-0.027	-0.067	-0.001	-0.196	0.042
.	0.000	0.786	0.639	0.411	0.042	0.115	.
1990:4 to 2005:4	0.297	0.013	0.000	-0.003	0.001	-0.261	0.052
.	0.085	0.791	0.995	0.978	0.685	0.074	.
2006:1 to 2019:4	0.143	0.003	0.015	-0.028	0.000	0.171	0.044
	0.452	0.973	0.896	0.858	0.879	0.574	.

For each sample period, the first row gives the parameter estimates, and the second row the p-value of the null that the coefficient equals zero, using heteroscedasticity-consistent standard errors. The measure of uncertainty is the cross-sectional median of the variance estimates from the individuals' current-year annual GDP growth histograms.

Table 5: Aggregate time-series rounding results for UR next year

	Constant	Q2	Q3	Q4	Time Tr.	Macro Unc.	R^2
Dep. var.: Proportion of forecasts rounded to M5 at each t							
1981:3 to 2019:4	0.290	-0.024	-0.026	-0.042	-0.001	0.000	0.305
.	0.000	0.146	0.163	0.011	0.000	0.983	.
1981:3 to 1990:3	0.237	-0.055	-0.040	-0.073	-0.001	0.050	0.179
.	0.015	0.131	0.380	0.077	0.768	0.256	.
1990:4 to 2019:4	0.319	-0.019	-0.027	-0.034	-0.001	-0.037	0.255
.	0.000	0.300	0.206	0.059	0.000	0.132	.
1990:4 to 2005:4	0.235	-0.033	-0.051	-0.045	0.000	-0.022	0.076
.	0.000	0.204	0.083	0.077	0.557	0.437	.
2006:1 to 2019:4	0.147	-0.004	-0.001	-0.021	0.000	-0.009	0.025
.	0.069	0.858	0.982	0.349	0.826	0.893	.
Dep. var.: Proportion relative to those reported to 1 decimal place							
1981:3 to 2019:4	0.221	-0.024	-0.022	-0.030	0.000	0.031	0.045
.	0.000	0.198	0.323	0.129	0.851	0.146	.
1981:3 to 1990:3	0.237	-0.055	-0.040	-0.073	-0.001	0.050	0.178
.	0.014	0.131	0.380	0.078	0.760	0.258	.
1990:4 to 2019:4	0.245	-0.020	-0.026	-0.026	0.000	-0.022	0.016
.	0.000	0.341	0.319	0.250	0.843	0.444	.
1990:4 to 2005:4	0.198	-0.036	-0.052	-0.040	0.001	-0.022	0.132
.	0.001	0.167	0.070	0.117	0.066	0.447	.
2006:1 to 2019:4	0.136	-0.005	0.006	-0.002	0.000	0.048	0.013
	0.277	0.882	0.902	0.951	0.647	0.665	.

For each sample period, the first row gives the parameter estimates, and the second row the p-value of the null that the coefficient equals zero, using heteroscedasticity-consistent standard errors. The measure of uncertainty is the cross-sectional median of the variance estimates from the individuals' next-year annual GDP growth growth histograms.

Table 6: Analysis of Rounding of CPI and UR Forecasts: Rank Correlation Tests across Individuals

Rounding and Accuracy			
CPI current	CPI next year	UR current	UR next year
0.068	0.120	0.043	0.160
0.768	0.905	0.677	0.959
Uncertainty and Rounding			
CPI current	CPI next year	UR current	UR next year
-0.064	0.098	-0.032	0.032
0.246	0.856	0.363	0.633
Accuracy and Uncertainty			
CPI current	CPI next year	UR current	UR next year
0.126	0.019	0.004	0.043
0.915	0.580	0.517	0.681

Accuracy in the first and second panels is measured by MSFE, and uncertainty in the second and third by the variance of the respondents' current or next year output growth or inflation histograms, normalized by the cross-sectional averages. Rounding is calculated as the proportion of the respondent's forecasts which are an exact multiple of 0.5.

Table 7: Reported probabilities of decline of SPF respondents, $p^h_{j,t}$, surveys 1981:3 to 2019:4

Forecast	Current quarter	1-quarter	2-quarter	3-quarter	4-quarter
0	0.212	0.096	0.058	0.054	0.057
5	0.168	0.153	0.130	0.111	0.118
10	0.144	0.190	0.202	0.199	0.179
15	0.054	0.091	0.127	0.128	0.112
20	0.061	0.100	0.136	0.153	0.156
25	0.033	0.054	0.063	0.085	0.095
30	0.030	0.046	0.062	0.066	0.069
35	0.013	0.015	0.018	0.024	0.026
40	0.027	0.029	0.028	0.024	0.030
45	0.009	0.008	0.005	0.006	0.007
50	0.023	0.030	0.021	0.017	0.022
55	0.002	0.003	0.004	0.003	0.001
60	0.009	0.010	0.008	0.004	0.004
65	0.003	0.004	0.001	0.000	0.000
70	0.007	0.006	0.002	0.001	0.001
75	0.008	0.008	0.003	0.001	0.001
80	0.010	0.006	0.002	0.001	0.001
85	0.003	0.002	0.000	0.000	0.000
90	0.013	0.004	0.000	0.001	0.000
95	0.006	0.001	0.000	0.000	0.000
100	0.019	0.004	0.000	0.000	0.000
Proportion reported as a multiple of :					
10 or 5	0.852	0.862	0.870	0.878	0.884
10	0.555	0.522	0.520	0.519	0.522
5 (excl. multiples 10)	0.298	0.340	0.350	0.358	0.362
5 (excl. 0)	0.640	0.766	0.813	0.824	0.827
No. Forecasts	4990	5067	5078	5075	5045

Notes. The table reports the proportion of probability of decline forecasts ($p^h_{j,t}$) reported as the value given in the first column, for $h = 0, 1, \ldots, 4$ across all respondents and surveys.

Table 8: Analysis of Probability of Decline Forecasts: Rank Correlation Tests across Individuals

Rounding of $p^h_{j,t}$ and Accuracy			
Rounding includes zero		Rounding excludes zero	
$h = 0$	$h = 4$	$h = 0$	$h = 4$
-0.065	0.154	0.596	0.361
0.241	0.953	1.000	1.000
Uncertainty and Rounding of $p^h_{j,t}$			
Rounding includes zero		Rounding excludes zero	
$h = 0$	$h = 4$	$h = 0$	$h = 4$
-0.248	-0.127	0.083	-0.020
0.003	0.084	0.816	0.414

Accuracy in the first panel is measured by normalized QPS. Uncertainty in the second panel by the variance of the respondents' current year output growth histograms, normalized by the cross-sectional average.

Table 9: Individual OLS regressions of normalised QPS score on a dummy for rounding of current-quarter and four-quarter ahead probability of decline forecasts, assuming zero-forecasts are not rounded

		Current-quarter		Four-quarter		Current QPS	+4 QPS
id	# Forecasts	Dummy	p-value	Dummy	p-value	$\hat{p}/p$	$\hat{p}/p$
1	2	3	4	5	6	7	8
421	105	0.491	0.042	0.576	0.006	0.746	0.763
426	105	0.880	0.049	0.292	0.456	0.868	0.588
428	70	0.625	0.105	0.445	0.022	0.663	0.873
433	99	0.374	0.018	0.238	0.072	0.918	1.096
84	98	0.539	0.000	0.373	0.097	1.004	1.048
411	71	0.689	0.000	0.135	0.088	0.967	1.702
446	89	0.867	0.000	0.033	0.883	0.866	0.643
65	67	1.490	0.017	2.080	0.000	0.725	0.916
484	89	-0.613	0.072	0.534	0.036	1.059	1.171
20	82	1.126	0.034	0.499	0.014	2.101	5.015
463	81	0.872	0.001	-0.146	0.395	0.969	0.740
510	78	3.169	0.046	0.823	0.002	0.927	1.549
407	79	0.653	0.003	-0.217	0.388	0.870	1.161
504	79	0.582	0.106	0.523	0.226	0.780	0.527
420	79	0.306	0.013	0.114	0.138	0.886	1.885
472	74	0.660	0.006	0.372	0.086	0.801	0.982
456	58	0.855	0.005	-0.011	0.959	1.097	1.210
518	71	0.521	0.150	-0.047	0.857	0.945	0.999
507	62	0.162	0.497	-0.047	0.834	0.933	0.902
99	65	4.578	0.032	-0.159	0.053	1.331	1.391
508	64	0.456	0.071	0.471	0.043	0.922	0.875
431	67	1.685	0.062	-0.251	0.189	0.757	0.778
422	60	0.796	0.052	0.077	0.447	0.860	1.262
506	46	0.793	0.000	0.168	0.579	1.010	1.113
512	59	2.177	0.001	1.572	0.000	0.721	0.772
483	56	1.345	0.242	0.007	0.988	0.754	0.491
549	56	0.245	0.250	0.083	0.745	0.887	0.964
546	56	0.564	0.015	-0.098	0.655	1.088	0.720
520	53	1.482	0.185	0.762	0.115	0.762	1.408
535	54	0.763	0.010	0.268	0.058	0.991	1.594
548	53	0.901	0.065	-0.121	0.598	1.014	0.880
524	52	1.933	0.001	0.598	0.008	0.890	0.904
527	48	0.697	0.037	-0.484	0.045	0.928	0.755
555	48	0.391	0.147	0.165	0.435	1.046	1.464
557	47	0.648	0.002	0.048	0.804	0.971	1.165
542	46	0.296	0.233	0.233	0.350	0.963	0.894
		0.944	22	0.275	10	27	20
		0.904	27	0.275	16	16	14

Rounding is defined as a multiple of 10, but excluding forecasts of zero.
The penultimate and last rows give the cross-sectional mean and standard deviations of coefficient estimates (columns 3 and 5), respectively, the number of p-values less than 0.05 and 0.01 (columns 4 and 6), and the number of ratios less than 1 and less than 0.9 (columns 7 and 8).
Of the 36 individual respondents making 40 or more forecasts, for 22 the dummy variable (rounding to 10) was statistically significant at the 5% level for the current quarter, and 10 for 4-quarters ahead.

Table 10: Individual OLS regressions of normalised QPS score on a dummy for rounding of current-quarter and four-quarter ahead probability of decline forecasts, assuming zero-forecasts are rounded

		Current-quarter		Four-quarter	
id	# Forecasts	Dummy	p-value	Dummy	p-value
1	2	3	4	5	6
421	105	0.491	0.042	0.576	0.006
426	105	0.794	0.075	0.292	0.456
428	70	-0.436	0.278	0.154	0.454
433	99	-0.228	0.085	0.083	0.550
84	98	0.144	0.189	0.373	0.097
411	71	0.104	0.291	0.135	0.088
446	89	-0.325	0.156	0.033	0.883
65	67	-2.179	0.124	0.528	0.292
484	89	-0.695	0.040	0.534	0.036
20	82	-0.041	0.856	-0.154	0.226
463	81	0.617	0.004	-0.146	0.395
510	78	-1.173	0.037	-0.030	0.909
407	79	0.412	0.035	-0.217	0.388
504	79	0.491	0.175	0.368	0.397
420	79	0.048	0.703	0.099	0.205
472	74	-0.150	0.285	0.203	0.329
456	58	-0.046	0.792	-0.011	0.959
518	71	0.521	0.150	-0.047	0.857
507	62	-0.365	0.095	-0.047	0.834
99	65	-0.182	0.558	-0.159	0.053
508	64	0.386	0.127	0.337	0.140
431	67	0.302	0.719	-0.320	0.093
422	60	0.354	0.306	0.041	0.682
506	46	0.399	0.030	0.168	0.579
512	59	1.196	0.042	1.479	0.001
483	56	1.345	0.242	0.007	0.988
549	56	0.000	1.000	0.083	0.745
546	56	0.564	0.015	-0.098	0.655
520	53	0.873	0.278	0.756	0.084
535	54	0.413	0.137	0.268	0.058
548	53	0.754	0.120	-0.121	0.598
524	52	1.546	0.009	0.598	0.008
527	48	-0.195	0.492	-0.484	0.045
555	48	0.148	0.596	-0.130	0.561
557	47	0.162	0.376	0.109	0.569
542	46	0.085	0.727	0.233	0.350
		0.170	9	0.153	5
		0.679	12	0.153	11

Rounding is defined as a multiple of 10, and a forecast of zero is assumed to denote rounding.
The penultimate and last rows give the cross-sectional mean and standard deviations of coefficient estimates (columns 3 and 5), respectively, the number of p-values less than 0.05 and 0.01 (columns 4 and 6).
Of the 36 individual respondents making 40 or more forecasts, for 9 the dummy variable (rounding to 10) was statistically significant at the 5% level for the current quarter, and 5 for 4-quarters ahead.

Table 11: Simulation results. The effects of rounding on event forecasting, on comparisons of true and rounded forecasts, and correcting rounded forecasts

			Forecasting Falls			Forecasting p^0 with $\hat{p}$		
Hetero.	σ_{ε_i}	$E\left(p^0 - 1_{w<0}\right)^2$	R_1	R_2	R_3	R_1	R_2	R_3
			Noisy Information					
0	1	0.134	1.003	1.001	1.003	0.269	0.116	0.269
0	1	0.132	1.000	0.999	1.000	0.257	0.106	0.257
0	1	0.136	1.005	1.002	1.005	0.279	0.128	0.279
1	1	0.148	1.002	1.001	1.002	0.852	1.912	1.038
1	1	0.134	0.999	0.999	0.999	0.150	0.154	0.165
1	1	0.155	1.005	1.003	1.005	4.698	12.905	7.153
0	3	0.154	1.002	1.001	1.002	0.336	0.170	0.336
0	3	0.154	1.000	0.999	1.000	0.318	0.153	0.318
0	3	0.155	1.005	1.003	1.005	0.350	0.185	0.350
1	3	0.158	1.003	1.001	1.002	0.387	0.254	0.382
1	3	0.154	1.001	1.000	1.000	0.337	0.181	0.331
1	3	0.159	1.004	1.003	1.004	0.477	0.495	0.500
			Private Information					
0	1	0.112	1.003	1.001	1.003	0.151	0.083	0.151
0	1	0.109	1.000	0.999	1.000	0.142	0.072	0.142
0	1	0.115	1.005	1.004	1.005	0.158	0.091	0.158
1	1	0.134	1.002	1.001	1.002	0.536	0.882	0.633
1	1	0.112	1.000	0.999	1.000	0.126	0.094	0.132
1	1	0.144	1.006	1.004	1.006	3.110	5.420	4.454
0	3	0.143	1.003	1.001	1.003	0.239	0.133	0.239
0	3	0.142	1.000	0.999	1.000	0.228	0.124	0.228
0	3	0.145	1.005	1.003	1.005	0.250	0.146	0.250
1	3	0.147	1.003	1.001	1.002	0.261	0.165	0.259
1	3	0.144	1.001	0.999	1.001	0.240	0.135	0.237
1	3	0.149	1.004	1.003	1.004	0.350	0.339	0.365
			Diagnostic Expectations					
0	1	0.120	1.003	1.001	1.003	0.170	0.085	0.170
0	1	0.118	1.001	0.999	1.001	0.162	0.078	0.162
0	1	0.123	1.005	1.003	1.005	0.180	0.095	0.180
1	1	0.154	1.003	1.001	1.003	0.620	1.016	0.753
1	1	0.120	1.000	0.999	1.001	0.079	0.075	0.087
1	1	0.180	1.005	1.003	1.005	3.865	7.191	5.863
0	3	0.182	1.003	1.001	1.003	0.135	0.099	0.135
0	3	0.178	1.001	0.999	1.001	0.128	0.088	0.128
0	3	0.188	1.004	1.004	1.004	0.141	0.109	0.141
1	3	0.255	1.003	1.004	1.003	0.100	0.139	0.100
1	3	0.181	1.001	1.000	1.001	0.083	0.086	0.083
1	3	0.312	1.005	1.006	1.004	0.196	0.221	0.208

See the Appendix for the details.
In each set of 3 rows, the first is the mean across respondents, and the second and third rows are the minimum and maximum across respondents.
A zero in the first column indicates agents are homogeneous, and a 1 indicates heterogeneity.
R_1, denotes rounding to a multiple of 0.1 with probability 0.4. R_2 makes the probability of rounding (to a multiple of 0.1) equal to p^0. R_3 sets the probability of rounding (to a multiple of 0.1) to (0.4 times) the ratio of the agent's forecast-error variance to the cross-sectional median.
The results for 'Forecasting Falls' are for the QPS scores for forecasting the event with a rounded probability, to the forecast using the true probability. The last three columns are the expected squared errors of the corrected forecasts to the rounded forecasts.

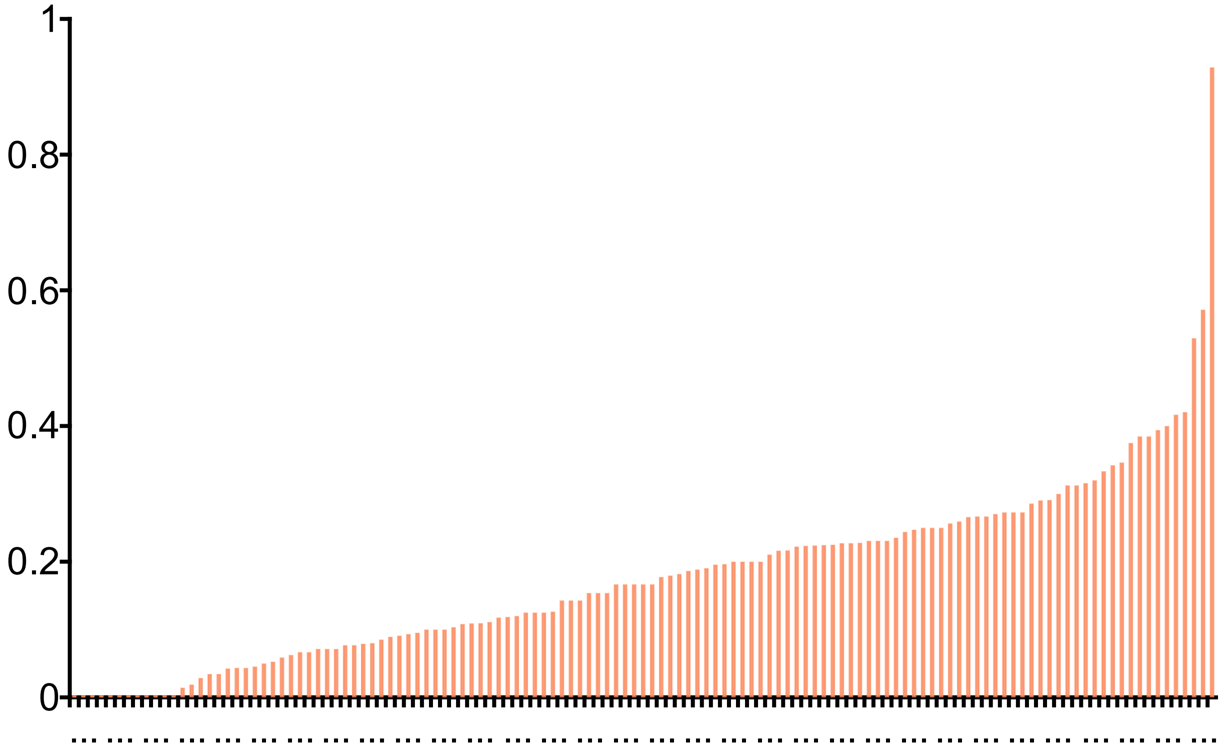

Figure 1: Proportion of each respondents CPI forecasts (current-year) which are a multiple of 0.5

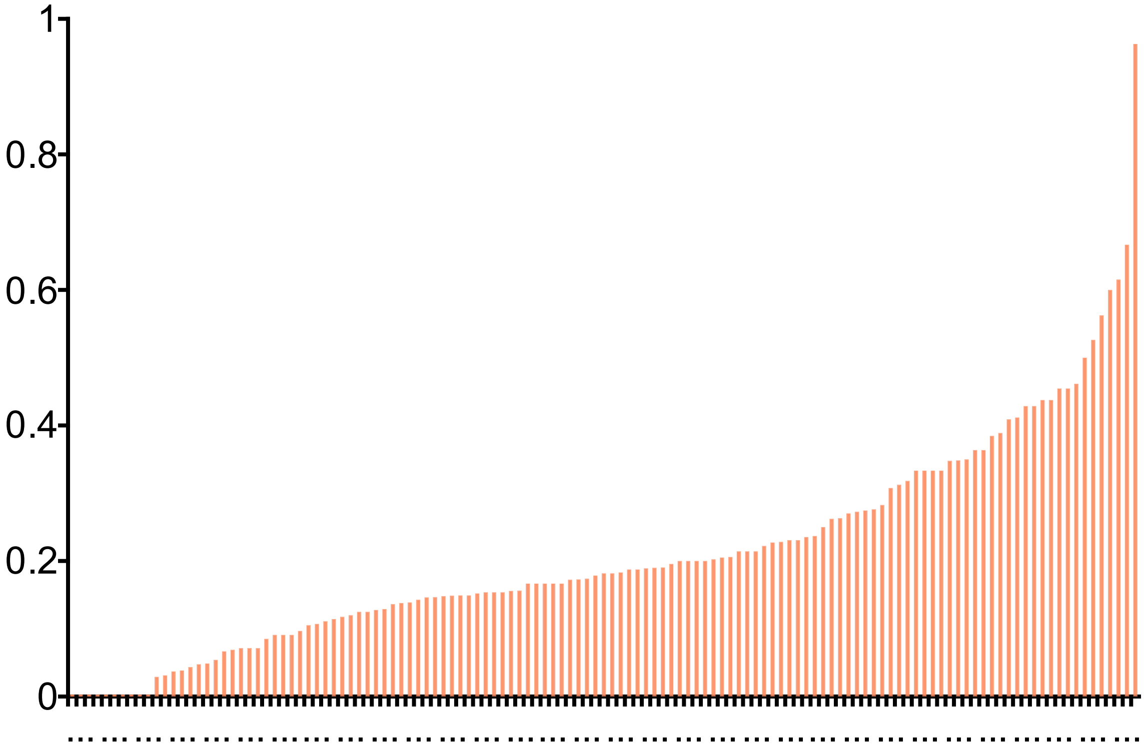

Figure 2: Proportion of each respondents CPI forecasts (next-year) which are a multiple of 0.5

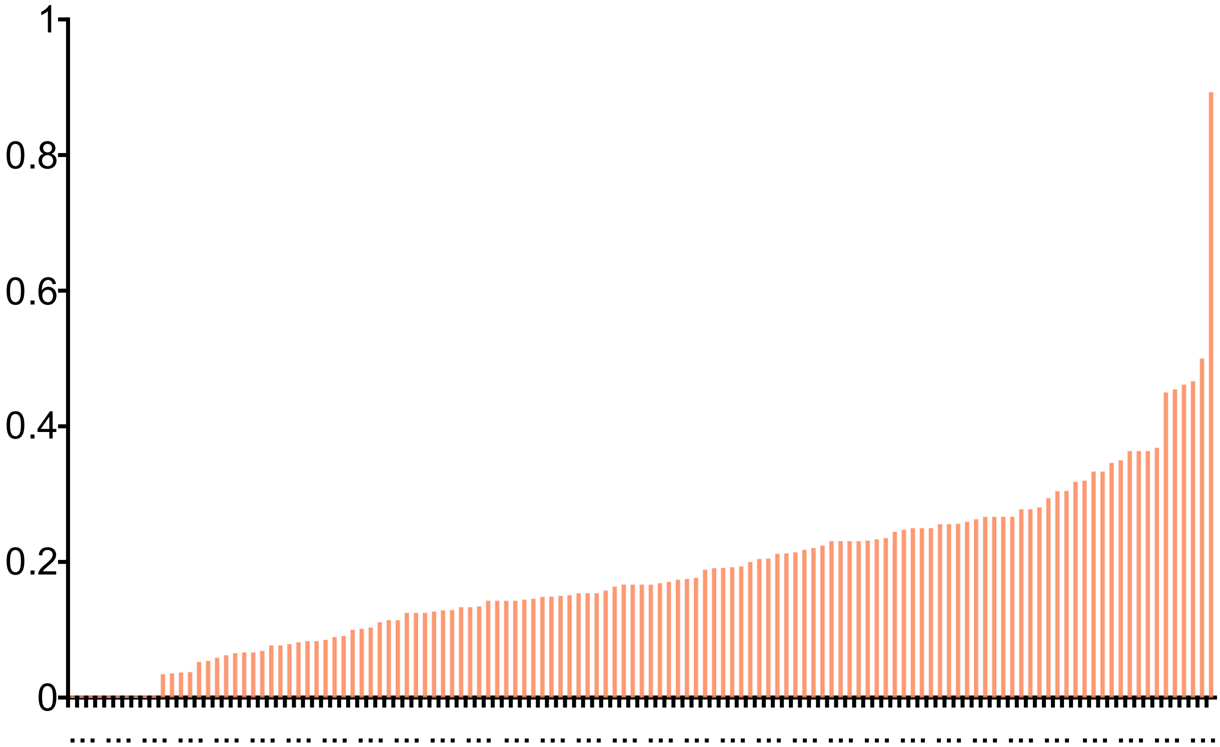

Figure 3: Proportion of each respondents UR forecasts (current-year) which are a multiple of 0.5

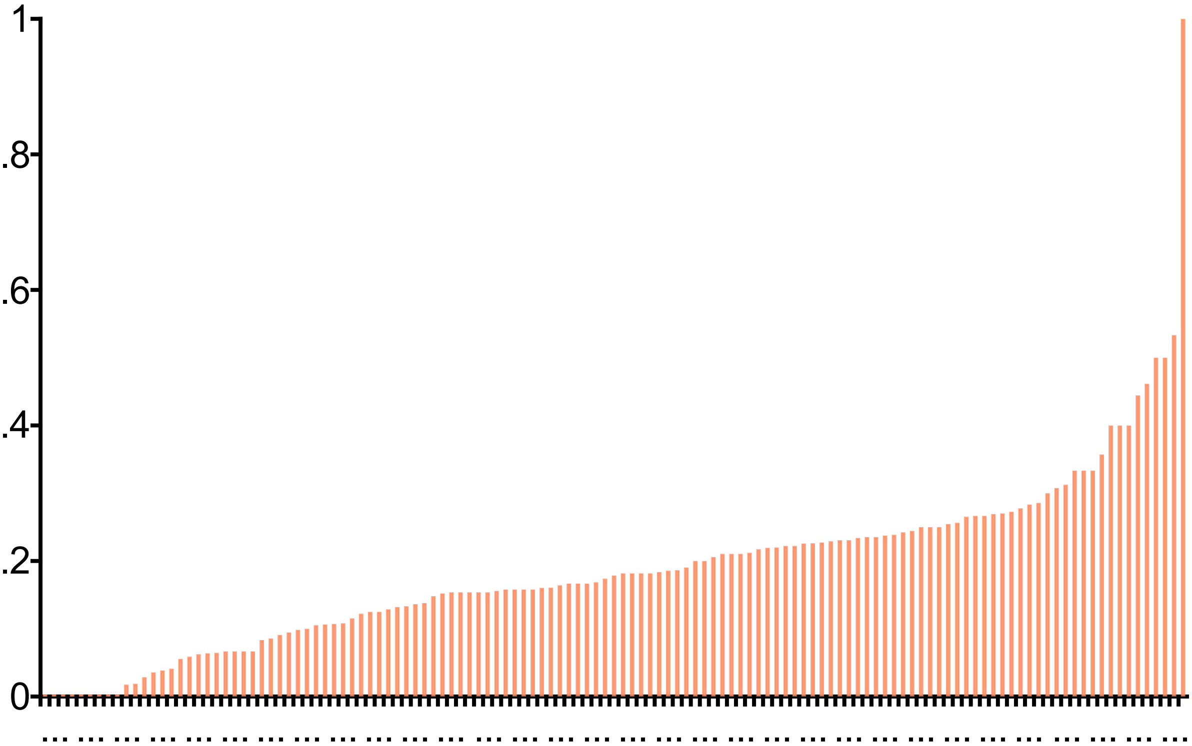

Figure 4: Proportion of each respondents UR forecasts (next-year) which are a multiple of 0.5

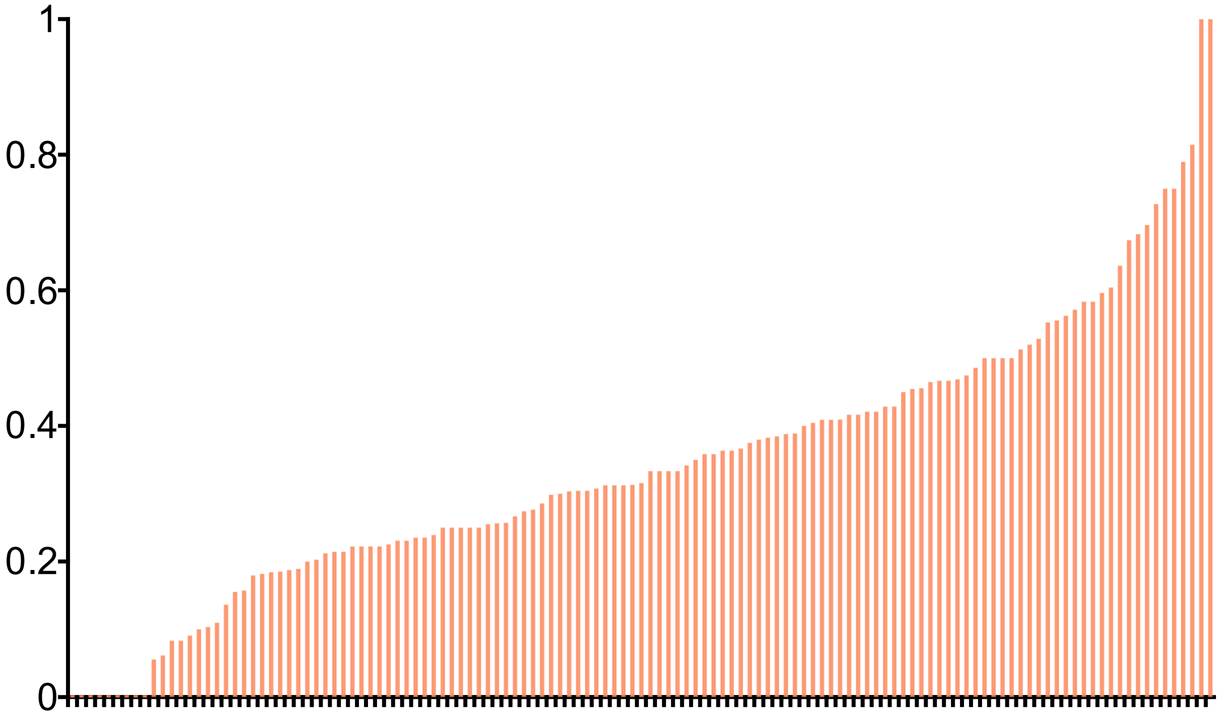

Figure 5: Proportion of each respondents current-quarter probabilities of decline which are rounded to 10, excluding zeros

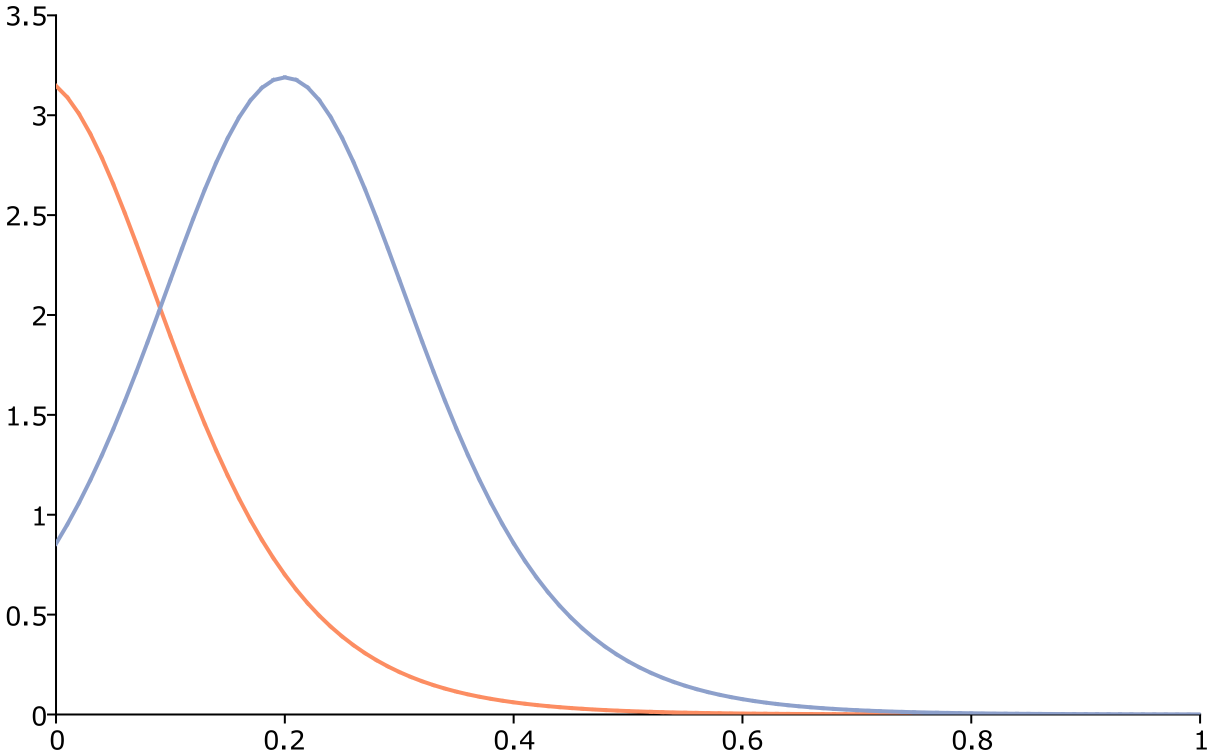

Figure 6: ϕ^l (left) and ϕ^h (right) densities in the model of the cross-sectional distribution of the current-quarter reported probabilities of decline (zeros assumed to not denote rounding). The estimated location and scale parameters are $\mu_l, \sigma_l, \mu_h, \sigma_h = -0.0181, 0.0784, 0.2001, 0.1237$.

6 Appendix to section 4.3

The data generation process for quarterly output growth in the simulations is an $AR(1)$:

$$y_t = \beta_0 + \beta y_{t-1} + \eta_t$$

where η_t is an iid gaussian innovation, $\eta_t \sim N(0, \sigma_\eta^2)$. Individual forecasters each receive a signal s_{it}:

$$s_{it} = y_t + \varepsilon_{it},$$

where $\varepsilon_{it} \sim N(0, \sigma_{\varepsilon_i}^2)$, and $\sigma_{\varepsilon_i}^2 = \sigma_\varepsilon^2 \; \forall_i$ signifies all agents receive the same quality signal, and are homogeneous.

In the base case of noisy information NI, agent i's information set at time t, $\mathcal{I}_{i,t} = \{s_{it}, s_{it,t-1}, \ldots\}$, the history of signals received by agent i through t. Past values of y are not observed. Under private information PI, $\mathcal{I}_{i,t}$ is augmented with y_{t-1}, $y_{t-2}, \ldots$.

Noisy Information (NI) and Diagnostic Expecations (DE)

The forecast of t that incorporates s_{it} is given by:

$$\begin{aligned} f_{it|t} &= K_i s_{it} + (1 - K_i) f_{it|t-1} &(5)\\ &= f_{it|t-1} + K_i \left(s_{it} - f_{it|t-1}\right). &(6)\end{aligned}$$

This updates the forecast of t based on information through $t-1$, $f_{it|t-1}$, optimally in a MMSE-sense. The optimal weight is given by $K_i = \Sigma_i / \left(\Sigma_i + \sigma_{\varepsilon_i}^2\right)$, where:

$$K_i = \frac{1}{2}\left(-(1-\beta)^2 \sigma_{\varepsilon_i}^2 + \sigma_\eta^2 + \sqrt{\left[(1-\beta)^2 \sigma_{\varepsilon_i}^2 - \sigma_\eta^2\right]^2 + 4\sigma_{\varepsilon_i}^2 \sigma_\eta^2}\right)$$

(see,e.g., Bordalo *et al.* (2018)). The 1-step forecast is calculated from (11), as $f_{i,t+1|t} = \beta_0 + \beta f_{it|t}$.

The steady-state variance of the forecast error is:

$$Var\left(y_t - f_{it|t} | \mathcal{I}_{i,t}\right) = \frac{\Sigma_i \sigma_{\varepsilon_i}^2}{\Sigma_i + \sigma_{\varepsilon_i}^2}$$

and:

$$Var\left(y_{t+1} - f_{it+1t} | \mathcal{I}_{i,t}\right) = \frac{\beta^2 \sigma_{\varepsilon_i}^2 \Sigma_i}{\sigma_{\varepsilon_i}^2 + \Sigma_i} + \sigma_\eta^2.$$

Under Diagnostic Expectations, (6) becomes:

$$f_{it|t} = f_{it|t-1} + (1+\theta) K_i \left(s_{it} - f_{it|t-1}\right). \tag{7}$$

where $\theta > 0$ indicates news is overweighted relative to the optimal amount given by the Kalman gain K.

Private Information (PI)

The rational expectations forecast for agent i under PI is given by:

$$\begin{aligned} f_{it|t} &= \lambda_i s_{it} + (1-\lambda_i)(\beta_0 + \beta y_{t-1}) && (8) \\ &= \beta_0 + \beta y_{t-1} + \lambda_i [s_{it} - (\beta_0 - \beta y_{t-1})] && (9) \\ &= \beta_0 + \beta y_{t-1} + \lambda_i \eta_t + \lambda \varepsilon_{it} && (10) \end{aligned}$$

where $\lambda_i = \sigma_\eta^2 \left(\sigma_\eta^2 + \sigma_{\varepsilon_i}^2\right)^{-1}$. That is, it optimally combines the model forecast of t based on information up to $t-1$, $\beta_0 + \beta y_{t-1}$, with the time t signal s_{it}. The 1-step forecast of $t+1$ is simply, as above:

$$f_{i,t+1|t} = \beta_0 + \beta f_{it|t} \tag{11}$$

and for h-steps ahead:

$$f_{i,t+h|t} = \frac{\beta_0 \left(1-\beta^h\right)}{1-\beta} + \beta^h f_{it|t}.$$

The variances of the forecast errors are:

$$\begin{aligned} Var\left(y_t - f_{it|t} | \mathcal{I}_{i,t}\right) &= \frac{\sigma_{\varepsilon_i}^2 \sigma_\eta^2}{\sigma_{\varepsilon_i}^2 + \sigma_\eta^2} && (12) \\ Var\left(y_{t+1} - f_{it+!|t} | \mathcal{I}_{i,t}\right) &= \frac{\beta^2 \sigma_{\varepsilon_i}^2 \sigma_\eta^2}{\sigma_{\varepsilon_i}^2 + \sigma_\eta^2} + \sigma_\eta^2. \end{aligned}$$

If we let f_{it} and σ_{it}^2 denote the forecast, and forecast-error variance for one of NI, DE and PI, for a given horizon, then forecast probabilities of decline are given by:

$$p_{it} = \Phi\left(\frac{-f_{it}}{\sigma_{it}}\right). \tag{13}$$

Calibration

The model is loosely calibrated on U.S. real quarterly GDP growth We suppose $\beta_0 = 0.50$, and $\beta = 0.36$. This repoduces the unconditional growth rate of quarterly real GDP of 0.78 for the period 1947:1 – 2018:2 (2018:3 data vintage). The AR(1) model estimated standard error is $\sigma_\eta = 0.88$. These values are used for β_0, β and σ_η throughout.

In the private information model, setting $\sigma_\varepsilon = 3$ approximately reproduces the (average over time) cross-sectional standard deviation in the current-quarter output growth forecasts (1992 – 2018) of 0.22 assuming homogeneous forecasters. Disagreement varies inversely with σ_ε, because higher σ_ε means less weight is given to private signals, which are the only source of disagreement. However, setting $\sigma_\varepsilon = 3$ when $\sigma_\eta = 0.88$ suggests a weight of less than one tenth on the signal, and we also experiment with $\sigma_\varepsilon = 1$.

Forecast heterogeneity is determined such that the standard deviations of the signals are evenly spaced from 1 to 3, when the signals are (relatively) informative, $\sigma_\varepsilon = 1$, and from 3 to 9 when $\sigma_\varepsilon = 3$.

For DE, we set $\theta = \frac{1}{2}$ - all other quantities under DE, such as the forecast-error variance, are unchanged relative to NI (see Bordalo *et al.* (2018)).